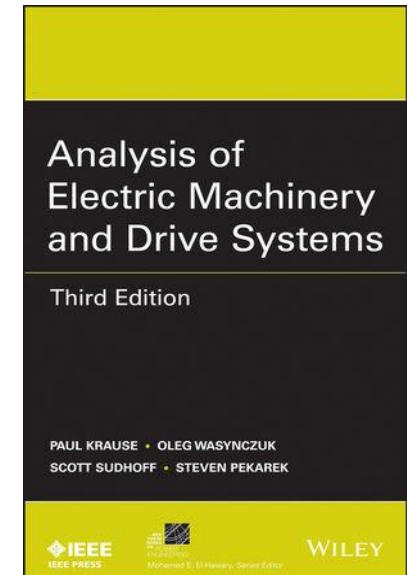
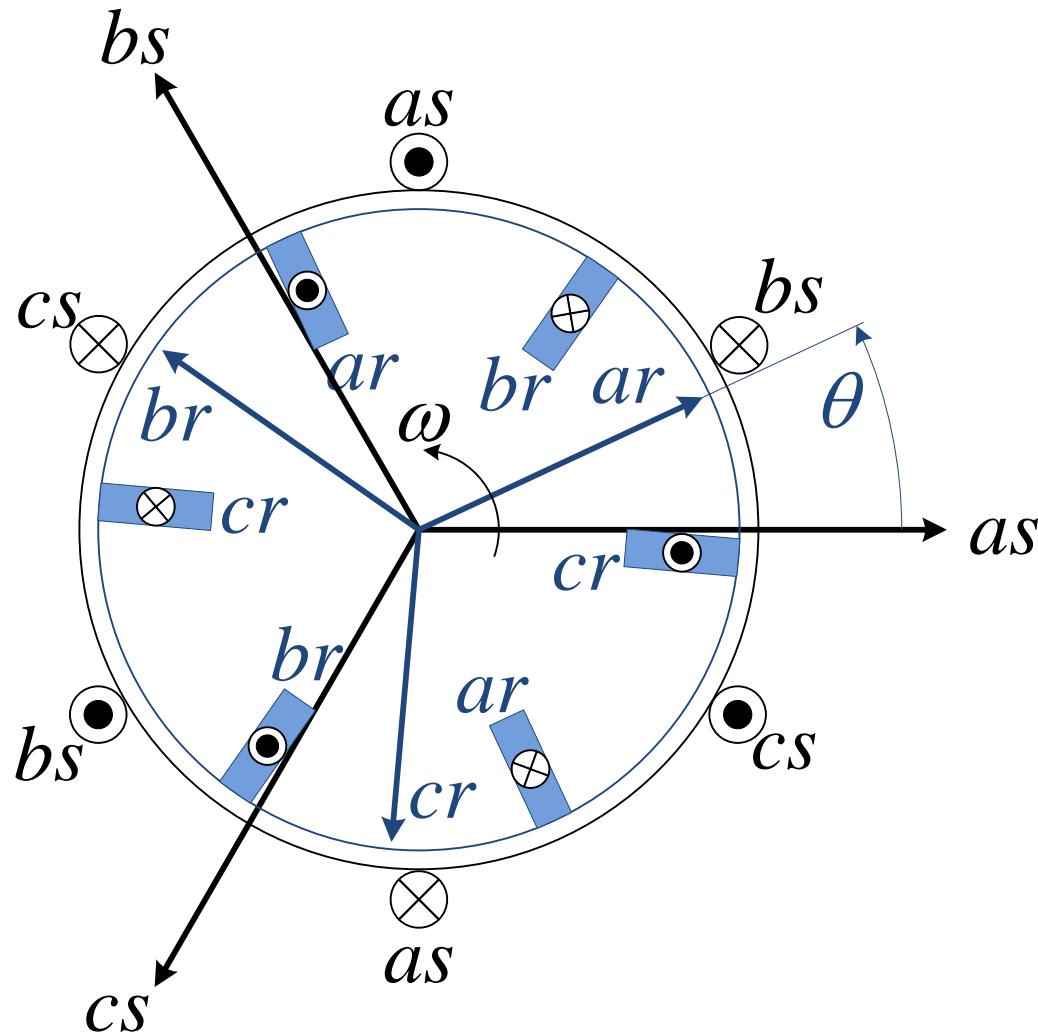


# DINAMIČKI MODEL (SIMETRIČNOG) TROFAZNOG ASINHRONOG MOTORA



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# Dinamički model asinhronog motora u faznom (abc) domenu

Naponska jednačina:  
(diferencijalna)

$$\vec{u}_{abcs} = \mathbf{R}_s \cdot \vec{i}_{abcs} + \frac{\partial}{\partial t}(\vec{\varphi}_{abcs})$$

$$\vec{u}'_{abcr} = \mathbf{R}'_r \cdot \vec{i}'_{abcr} + \frac{\partial}{\partial t}(\vec{\varphi}'_{abcr})$$

Jednačina flukseva:  
(algebarska)

$$\begin{bmatrix} \vec{\varphi}_{abcs} \\ \vec{\varphi}'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \vec{i}_{abcs} \\ \vec{i}'_{abcr} \end{bmatrix}$$

U prethodnim jednačinama koristi se vektorski zapis faznih valičina:

$$\vec{f}_{abc} = [f_a? \quad f_b? \quad f_c?]^T$$

$$\mathbf{R}_s = R_s \cdot \mathbf{I} \qquad \qquad \mathbf{R}'_r = R'_r \cdot \mathbf{I}$$

Matrice induktivnosti:

$$\mathbf{L}_s = \begin{bmatrix} \lambda_s + M_s & -0,5M_s & -0,5M_s \\ -0,5M_s & \lambda_s + M_s & -0,5M_s \\ -0,5M_s & -0,5M_s & \lambda_s + M_s \end{bmatrix}$$

$$\mathbf{L}'_r = \begin{bmatrix} \lambda'_r + M_s & -0,5M_s & -0,5M_s \\ -0,5M_s & \lambda'_r + M_s & -0,5M_s \\ -0,5M_s & -0,5M_s & \lambda'_r + M_s \end{bmatrix}$$

Ako uvedemo smenu:

$$\alpha = \frac{2\pi}{3}$$

Matrica međusobne induktivnosti statora i rotora:

$$\mathbf{L}'_{sr} = M_s \cdot \begin{bmatrix} \cos \theta & \cos(\theta + \alpha) & \cos(\theta - \alpha) \\ \cos(\theta - \alpha) & \cos \theta & \cos(\theta + \alpha) \\ \cos(\theta + \alpha) & \cos(\theta - \alpha) & \cos \theta \end{bmatrix}$$

Posle svođenja "rotora na stator" jednačine za fluks i naponske jednačina su:

$$\begin{bmatrix} \vec{\varphi}_{abcs} \\ \vec{\varphi}'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \vec{i}_{abcs} \\ \vec{i}'_{abcr} \end{bmatrix}$$

$$\begin{bmatrix} \vec{u}_{abcs} \\ \vec{u}'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_s + p\mathbf{L}_s & p\mathbf{L}'_{sr} \\ p(\mathbf{L}'_{sr})^T & \mathbf{R}'_r + p\mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \vec{i}_{abcs} \\ \vec{i}'_{abcr} \end{bmatrix}$$

$$p = \frac{\partial}{\partial t} \quad - \text{operator diferenciranja}$$

Operator diferenciranja prikazan u jednačinama u matričnoj formi se odnosi na proizvod matrice induktivnosti i vektora struja.

# JEDNAČINA MOMENTA

Na osnovu relacija koje važe za elektro-mehaničku konverziju energije može se napisati izraz za električnu energiju koja se pretvara u mehaničku:

$$W_e = \frac{1}{2} (\vec{i}_{abcs})^T (\mathbf{L}_s - \lambda_s \cdot \mathbf{I}) \cdot \vec{i}_{abcs} + (\vec{i}_{abcs})^T \cdot \mathbf{L}'_{sr} \cdot \vec{i}'_{abcr} + \frac{1}{2} (\vec{i}'_{abcr})^T (\mathbf{L}'_r - \lambda'_r \cdot \mathbf{I}) \cdot \vec{i}'_{abcr}$$

Mehanička snaga motora može se izraziti preko elektromagnetskog momenta i brzine obrtanja:

$$\frac{\partial}{\partial t} W_e = m_e \cdot \frac{\partial}{\partial t} \theta_m$$

$\theta_m$  - stvarni mehanički položaj rotora.

$$\theta = P \cdot \theta_m$$

$\theta$  - položaj rotora izražen u el.rad/s.

$$\frac{\partial}{\partial t} W_e = m_e \cdot \frac{1}{P} \cdot \frac{\partial}{\partial t} \theta$$

Elektromagnetni moment motora je:

$$m_e = P \cdot \frac{\partial W_e}{\partial \theta} = P \cdot (\vec{i}_{abcs})^T \cdot \frac{\partial}{\partial \theta} [\mathbf{L}'_{sr}] \cdot \vec{i}'_{abcr}$$

---

$$m_e = -P \cdot M_s \cdot \left\{ \begin{aligned} & \left[ i_{as} \cdot \left( i'_{ar} - \frac{1}{2} i'_{br} - \frac{1}{2} i'_{cr} \right) + i_{bs} \cdot \left( -\frac{1}{2} i'_{ar} + i'_{br} - \frac{1}{2} i'_{cr} \right) + i_{cs} \cdot \left( -\frac{1}{2} i'_{ar} - \frac{1}{2} i'_{br} + i'_{cr} \right) \right] \cdot \sin \theta + \\ & + \frac{\sqrt{3}}{2} \left[ i_{as} \cdot (i'_{br} - i'_{cr}) + i_{bs} \cdot (i'_{cr} - i'_{ar}) + i_{cs} \cdot (i'_{ar} - i'_{br}) \right] \cdot \cos \theta \end{aligned} \right\}$$

Dobijeni izraz je veoma komplikovan i praktično neupotrebljiv.

# TRASFORMACIJA KOORDINATA

- U cilju uprošćenja analize uvodi se novi *REFERENTNI q-d-0 - sistem* koji može imati proizvoljnu brzinu. Prelazak iz realnog *abc* - sistema u *qd0* - sistem vrši se pomoću matrice transformacije *K*.
- Izborom brzine referentnog sistema postižu se jednostavnije analize prelaznih procesa.

# Izbor referentnog sistema

- **Stacionarni referentni sistem**

obezbeđuje rasprezanje namotaja mašine, čime se pojednostavljuje matrica induktivnosti.

$$\omega_{rs} = 0$$

*q-d stac.  
(α-β)*

- **Sinhrono rotirajući referentni sistem**

pored rasprezanja koordinata, oslobađa matricu induktivnosti zavisnosti od ugla rotora, odnosno vremena

$$\omega_{rs} = \omega_s$$

*q-d  
sinhrono rot.*

- **Referentni sistem vezan za rotor**

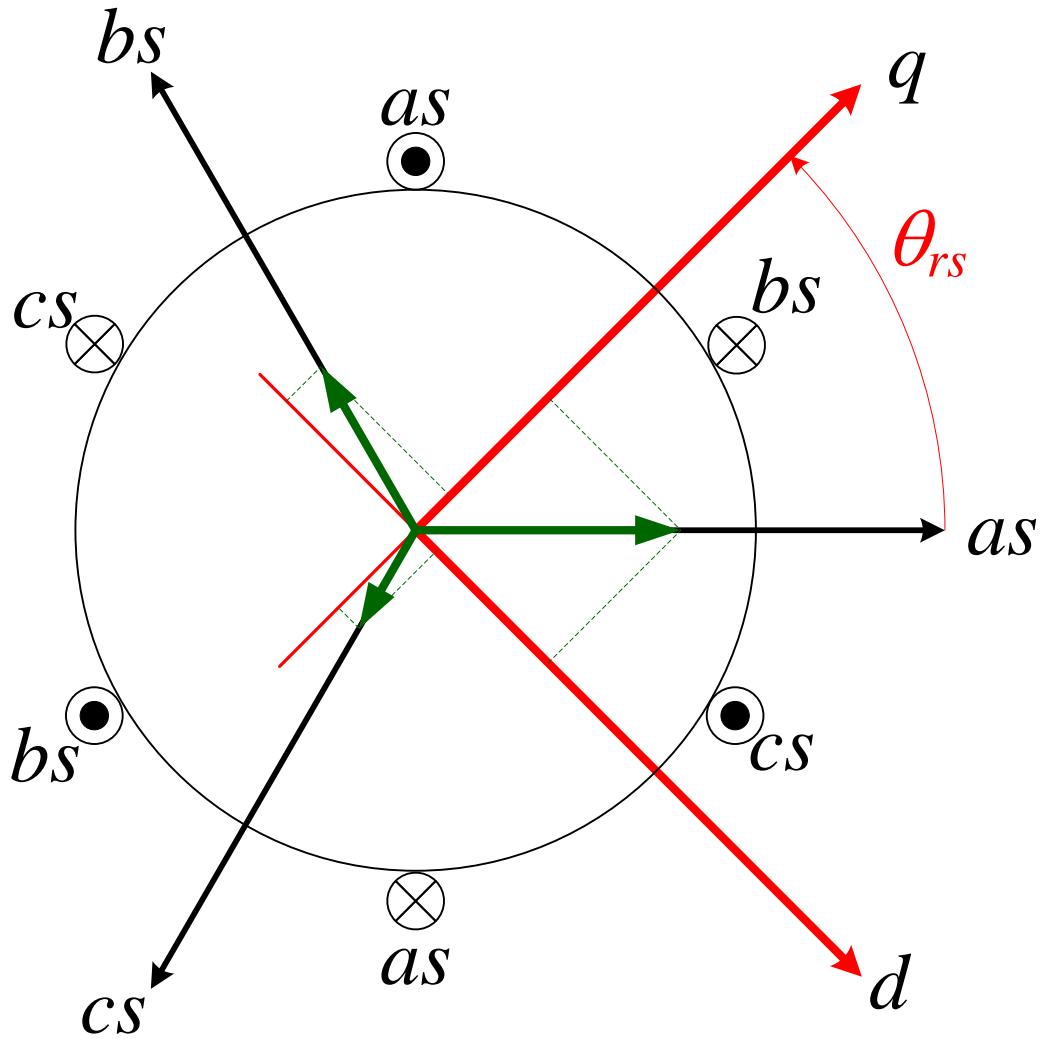
pruža pogodnosti analize mašina sa dvostranim napajanjem.

$$\omega_{rs} = \omega$$

*q-d  
rotorski*

U slučaju simetričnog sistema, nulta komponenta je nula, u svim referentnim sistemima.

# Transformacije statorskih veličina



$$\vec{f}_{qd0s} = \mathbf{K}_s \cdot \vec{f}_{abcs}$$

$$\vec{f}_{abcs} = [f_{as} \quad f_{bs} \quad f_{cs}]^T$$

$$\vec{f}_{qd0s} = [f_{qs} \quad f_{ds} \quad f_{0s}]^T$$

# Matrice transformacije

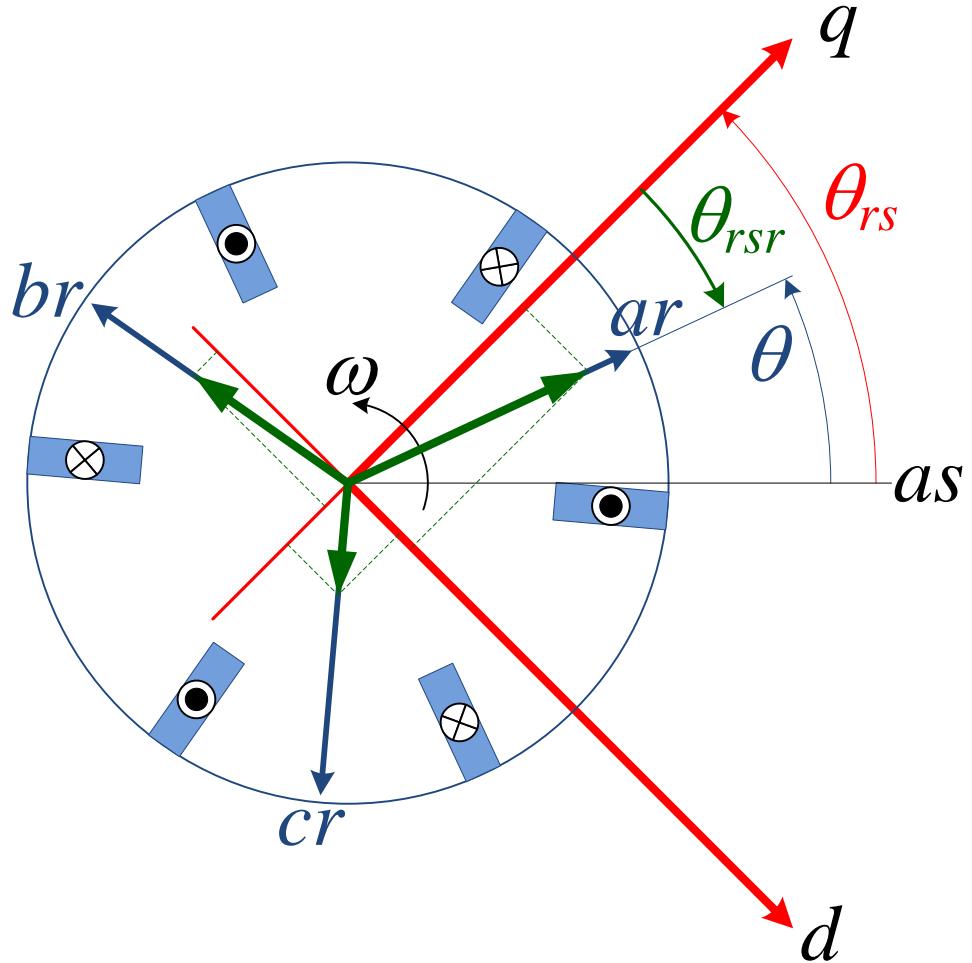
$$\mathbf{K}_s = \frac{2}{3} \begin{bmatrix} \cos \theta_{rs} & \cos(\theta_{rs} - \alpha) & \cos(\theta_{rs} + \alpha) \\ \sin \theta_{rs} & \sin(\theta_{rs} - \alpha) & \sin(\theta_{rs} + \alpha) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_s^{-1} = \begin{bmatrix} \cos \theta_{rs} & \sin \theta_{rs} & 1 \\ \cos(\theta_{rs} - \alpha) & \sin(\theta_{rs} - \alpha) & 1 \\ \cos(\theta_{rs} + \alpha) & \sin(\theta_{rs} + \alpha) & 1 \end{bmatrix}$$

$$\theta_{rs}(t) = \int_0^t \omega_{rs}(\xi) d\xi + \theta_{rs}(0)$$

Koeficijent transformacije 2/3 obezbeđuje invarijantnost po impedansi.

# Transformacije rotorskih veličina



Trenutni položaj rotora u odnosu na referentni sistem.

$$\theta_{rsr} = \theta_{rs} - \theta$$

$$\vec{f}'_{qd0r} = \mathbf{K}_r \vec{f}'_{abcr}$$

$$\vec{f}'_{abcr} = [ f'_{ar} \quad f'_{br} \quad f'_{cr} ]^T$$

$$\vec{f}'_{qd0r} = [ f'_{qr} \quad f'_{dr} \quad f'_{0r} ]^T$$

# Matrice transformacije

$$\mathbf{K}_r = \frac{2}{3} \cdot \begin{bmatrix} \cos \theta_{rsr} & \cos(\theta_{rsr} - \alpha) & \cos(\theta_{rsr} + \alpha) \\ \sin \theta_{rsr} & \sin(\theta_{rsr} - \alpha) & \sin(\theta_{rsr} + \alpha) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_r^{-1} = \begin{bmatrix} \cos \theta_{rsr} & \sin \theta_{rsr} & 1 \\ \cos(\theta_{rsr} - \alpha) & \sin(\theta_{rsr} - \alpha) & 1 \\ \cos(\theta_{rsr} + \alpha) & \sin(\theta_{rsr} + \alpha) & 1 \end{bmatrix}$$

$$\theta_{rs}(t) = \int_0^t \omega_{rs}(\xi) d\xi + \theta_{rs}(0) \quad \theta(t) = \int_0^t \omega(\xi) d\xi + \theta(0)$$

# Korišćene oznake

$$\alpha = \frac{2\pi}{3}$$

$\theta_{rs}$  - trenutni položaj referentnog sistema,

$\theta$  - trenutni položaj rotora motora,

$\omega_{rs}$  - brzina referentnog sistema,

$\omega$  - brzina motora,

$\omega_s$  - sinhrona brzina.

# Stacionarni koordinatni sistem

Kada je  $\omega_{rs}=0$ ,  $\theta_{rs}(0)=0$  i  $\alpha=\frac{2\pi}{3}$ ,

$$\theta_{rs} = \int_0^t 0 \cdot d\xi + \theta_{rs}(0) = 0,$$

$$\mathbf{K}_s = \frac{2}{3} \cdot \begin{bmatrix} \cos 0 & \cos\left(0 - \frac{2\pi}{3}\right) & \cos\left(0 + \frac{2\pi}{3}\right) \\ \sin 0 & \sin\left(0 - \frac{2\pi}{3}\right) & \sin\left(0 + \frac{2\pi}{3}\right) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$



Edith Clarke  
1883 - 1959

# Stacionarni koordinatni sistem

## Matrice transformacije statorskih veličina

$$\mathbf{K}_s = \frac{2}{3} \cdot \begin{bmatrix} 1 & -0,5 & -0,5 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_s^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -0,5 & -\frac{\sqrt{3}}{2} & 1 \\ -0,5 & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

# Stacionarni koordinatni sistem

## Matrice transformacije rotorskih veličina

$$\mathbf{K}_r = \frac{2}{3} \cdot \begin{bmatrix} \cos(-\theta) & \cos(-\theta - \alpha) & \cos(-\theta + \alpha) \\ \sin(-\theta) & \sin(-\theta - \alpha) & \sin(-\theta + \alpha) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_r^{-1} = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 1 \\ \cos(-\theta - \alpha) & \sin(-\theta - \alpha) & 1 \\ \cos(-\theta + \alpha) & \sin(-\theta + \alpha) & 1 \end{bmatrix}$$

# Šta se postiže ovom transformacijom?

## Statorske veličine

Primer simetričnog trofaznog sistema koji ima konstantnu učestanost:

$$f_{as} = f_{\max s} \cdot \cos(\omega_s \cdot t + \theta_s(0))$$

$$f_{bs} = f_{\max s} \cdot \cos(\omega_s \cdot t - \alpha + \theta_s(0))$$

$$f_{cs} = f_{\max s} \cdot \cos(\omega_s \cdot t + \alpha + \theta_s(0))$$

posle transformacije se dobija:

$$f_{qs} = f_{\max s} \cdot \cos(\omega_s \cdot t + \theta_s(0)) = f_{\alpha s}$$

$$f_{ds} = -f_{\max s} \cdot \sin(\omega_s \cdot t + \theta_s(0)) = -f_{\beta s}$$

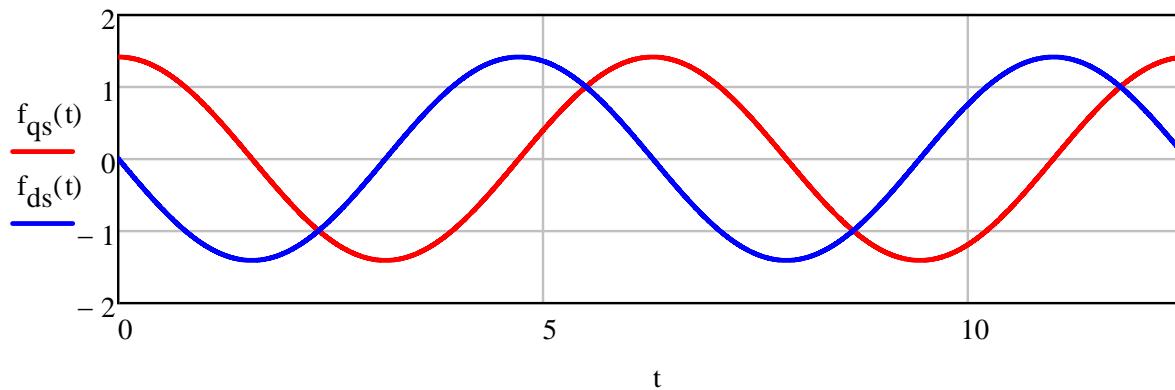
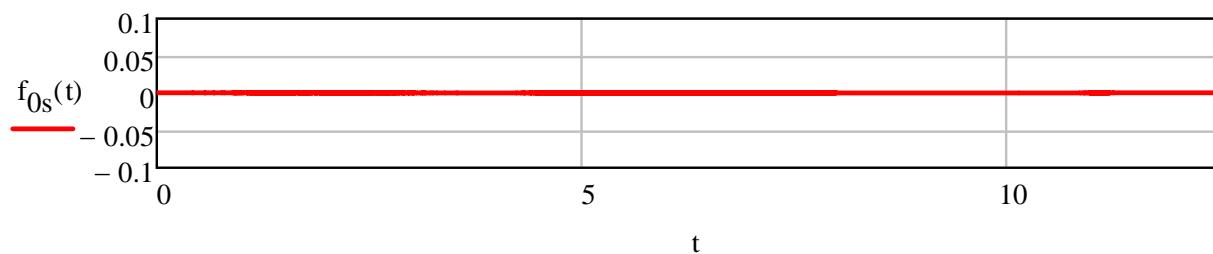
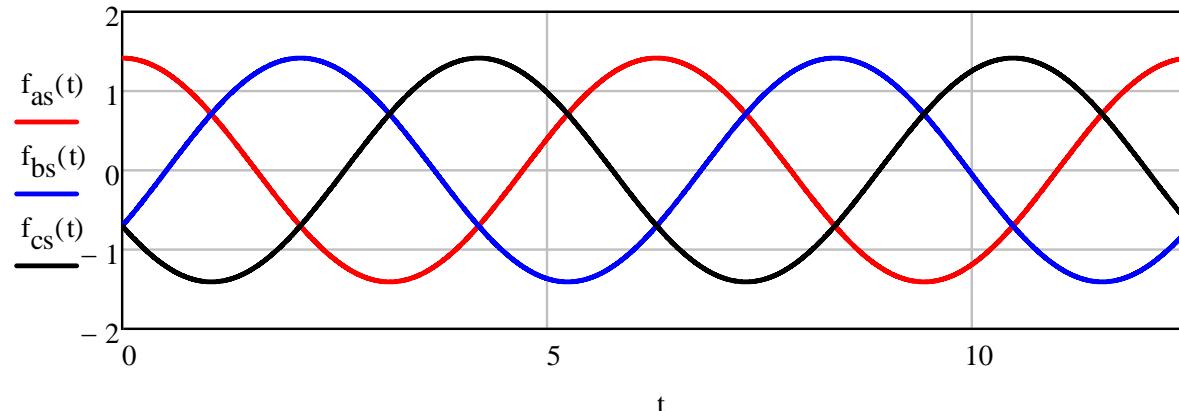
$$f_{0s} = 0 = \text{const.}$$

$$f_{\max s} = \sqrt{f_{qs}^2 + f_{ds}^2}$$

Umesto trofaznog naizmeničnog sistema dobijamo dvofazni sistem.

# Statorske veličine $\omega_{rs}=0$

Na graficima  
 $\omega_s=1$



# Šta se postiže ovom transformacijom?

## Rotorske veličine

Kada je  $\omega_{rs}=0$ ,  $\theta_{rs}(0)=0$  i  $\theta_{rsr}=0-\theta=-\theta$  za simetričan rotorski sistem:

$$f'_{ar} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0)]$$

$$f'_{br} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0) - \alpha]$$

$$f'_{cr} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0) + \alpha]$$

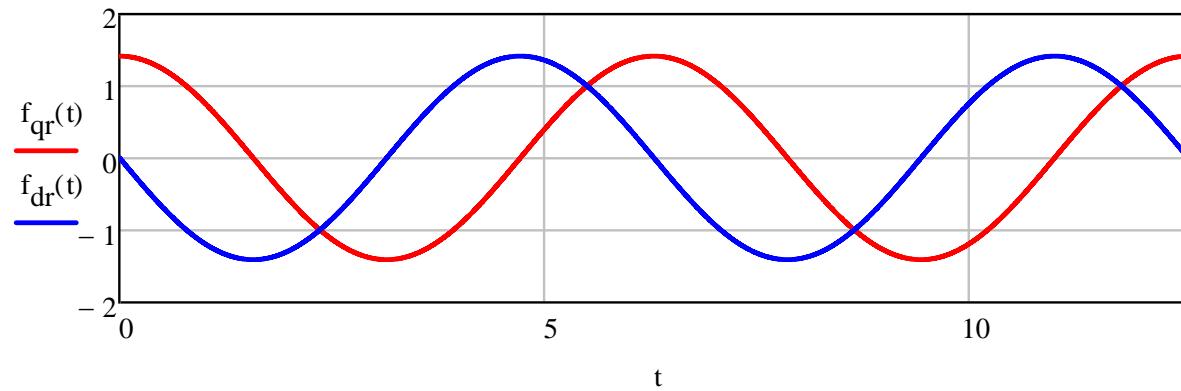
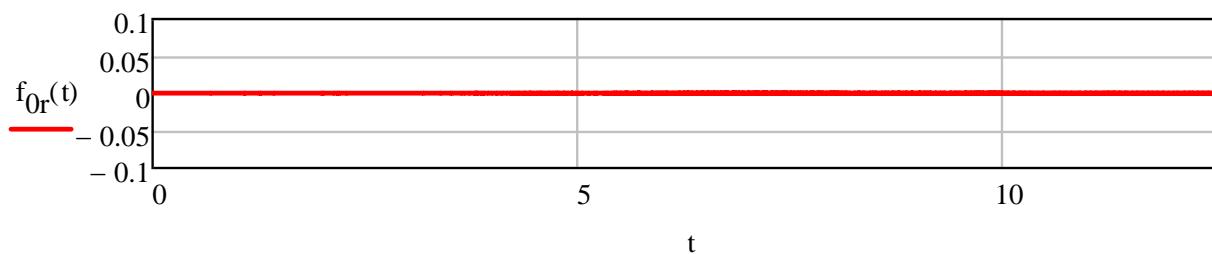
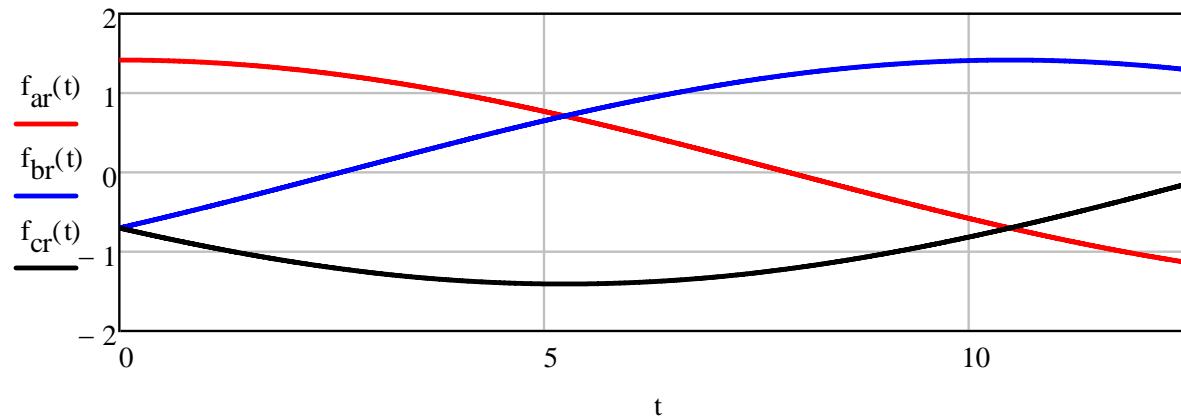
posle transformacije dobija se:

$$f'_{qr} = f'_{\max r} \cdot \cos(\omega_s \cdot t + \theta_r(0)) = f'_{\alpha r}$$

$$f'_{dr} = -f'_{\max r} \cdot \sin(\omega_s \cdot t + \theta_r(0)) = -f'_{\beta r}$$

$$f'_{0r} = 0$$

# Rotorske veličine $\omega_{rs}=0$

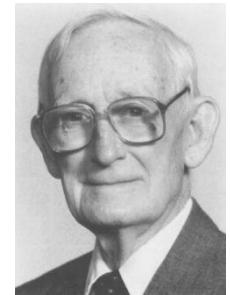


# Sinhrono rotirajući koordinatni sistem

Kada je  $\omega_{rs} = \omega_s$ ,  $\theta_{rs}(0) = 0$ ,  $\theta_s(0) = 0$  i  $\alpha = \frac{2\pi}{3}$ ,

$$\theta_{rs} = \theta_s = \int_0^t \omega_s(\xi) \cdot d\xi + \theta_s(0)$$

$$\mathbf{K}_s = \frac{2}{3} \cdot \begin{bmatrix} \cos \theta_s & \cos\left(\theta_s - \frac{2\pi}{3}\right) & \cos\left(\theta_s + \frac{2\pi}{3}\right) \\ \sin \theta_s & \sin\left(\theta_s - \frac{2\pi}{3}\right) & \sin\left(\theta_s + \frac{2\pi}{3}\right) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$



*Robert H. Park*  
Robert H. Park  
1902-1994

## Two-Reaction Theory of Synchronous Machines

## **eed Method of A BY B. H. PARK**

**Synopsis.**—Starting with the basic assumption of no saturation in  $\sigma$  vs.  $E$ , we find that the distribution of ionization energy  $\epsilon$  in  $J(\epsilon)$  is given by a formula which is identical with that for isotropic photoconductive dependent systems, except, however, that  $J(\epsilon)$  depends on  $\epsilon$  instead of  $E$ . The theory is applied to the case of  $\text{Ag}_2\text{S}$ .

present, and torque under steady and transient load conditions, especially detailed formulas are also developed which permit the determination of current and voltage waveforms when direct, alternating, and transient, as well as small deviations from no-load, normally stable, are considered.

**T**HIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Dubetz [1] to include the case of three-phase systems.

— and Nickle, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Then there is

$$\begin{aligned} e_1 &= p \psi_1 - \tau z_1 \\ e_2 &= p \psi_2 - \tau z_2 \end{aligned}$$

idealization is resort to, to the extent that saturation and hysteresis in every magnetic circuit need only be taken into account.

$$= -\frac{v_0}{3} [x_0 + t_0 + t_0] = -\frac{v_0}{3} \left[ x_0 - \frac{v_0 \sin \theta}{g} \right]$$

$$= -\frac{x_0 - x_0}{3} [i_1 \cos 2\theta + i_1 \cos (2\theta - 120^\circ)]$$

$$Q_{\text{L}} = I_L \cos(\theta - 120) - I_S \sin(\theta - 120) + i_s \cos(2\theta + 120)$$

$$= -\frac{i_s + i_b + i_c}{3} - \frac{\sigma_0 + \sigma_2}{3} \left[ \frac{1}{2} - \frac{i_s + i_c}{3} \right]$$

currents in the armature iron are neglected, and is

$$\begin{aligned} \text{the assumption that, as far as concerns effects depending on the position of the rotor, each successive winding may be regarded as, in effect, sinusoidally distributed.}^2 \\ A. \quad \text{Primary Current Equations} \\ \text{Introducing the primary current components } I_1 = I_1 \cos(\theta + 120^\circ) \quad + i_1 \sin(\theta + 120^\circ) \quad (2) \\ I_2 = I_2 \cos(\theta + 120^\circ) \quad - i_2 \sin(\theta + 120^\circ) \quad - I_3 \sin(\theta + 120^\circ) - z_3 \quad i_3 + i_4 + i_5. \end{aligned}$$

<sup>1</sup>Second stage, Dept. of General Electric Company, Schenectady, N. Y.

machines with one plane open measured.  
please for a machine with viscosity field constant.  
For more information see Bibliography.  
*Proceedings of the Winter Convention of the A. S. C. E., New York,  
N. Y., Jan. 26-Feb. 1, 1950.*

# Sinhrono rotirajući koordinatni sistem

## Matrice transformacije statorskih veličina

$$\mathbf{K}_s = \frac{2}{3} \begin{bmatrix} \cos \theta_s & \cos(\theta_s - \alpha) & \cos(\theta_s + \alpha) \\ \sin \theta_s & \sin(\theta_s - \alpha) & \sin(\theta_s + \alpha) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_s^{-1} = \begin{bmatrix} \cos \theta_s & \sin \theta_s & 1 \\ \cos(\theta_s - \alpha) & \sin(\theta_s - \alpha) & 1 \\ \cos(\theta_s + \alpha) & \sin(\theta_s + \alpha) & 1 \end{bmatrix}$$

# Sinhrono rotirajući koordinatni sistem

## Matrice transformacije rotorskih veličina

$$\mathbf{K}_r = \frac{2}{3} \cdot \begin{bmatrix} \cos(\theta_s - \theta) & \cos(\theta_s - \theta - \alpha) & \cos(\theta_s - \theta + \alpha) \\ \sin(\theta_s - \theta) & \sin(\theta_s - \theta - \alpha) & \sin(\theta_s - \theta + \alpha) \\ 0,5 & 0,5 & 0,5 \end{bmatrix}$$

$$\mathbf{K}_r^{-1} = \begin{bmatrix} \cos(\theta_s - \theta) & \sin(\theta_s - \theta) & 1 \\ \cos(\theta_s - \theta - \alpha) & \sin(\theta_s - \theta - \alpha) & 1 \\ \cos(\theta_s - \theta + \alpha) & \sin(\theta_s - \theta + \alpha) & 1 \end{bmatrix}$$

# Šta se postiže ovom transformacijom?

## Statorske veličine

Primer simetričnog trofaznog sistema koji ima konstantnu učestanost:

$$f_{as} = f_{\max s} \cdot \cos(\omega_s \cdot t + \theta_s(0))$$

$$f_{bs} = f_{\max s} \cdot \cos(\omega_s \cdot t - \alpha + \theta_s(0))$$

$$f_{cs} = f_{\max s} \cdot \cos(\omega_s \cdot t + \alpha + \theta_s(0))$$

posle transformacije se dobija:

$$f_{qs} = f_{\max s} \cdot \cos(\theta_s(0))$$

$$f_{ds} = -f_{\max s} \cdot \sin(\theta_s(0))$$

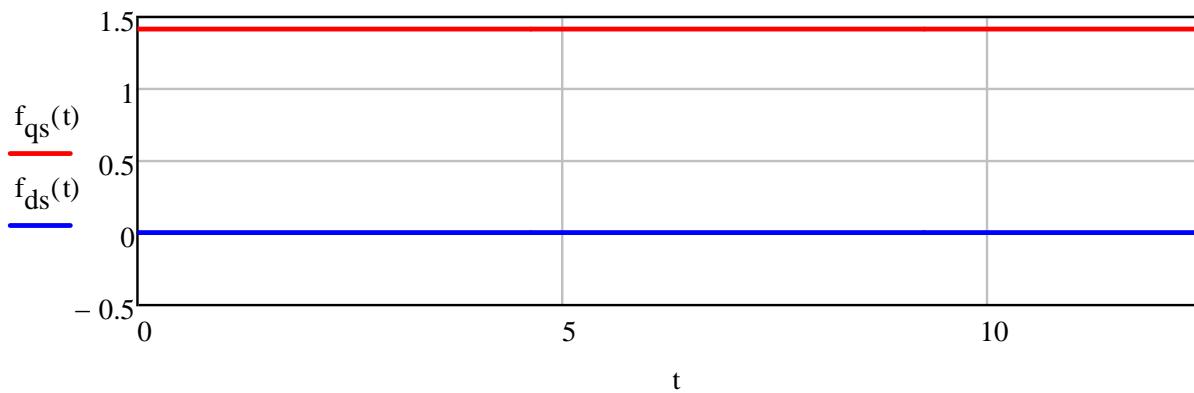
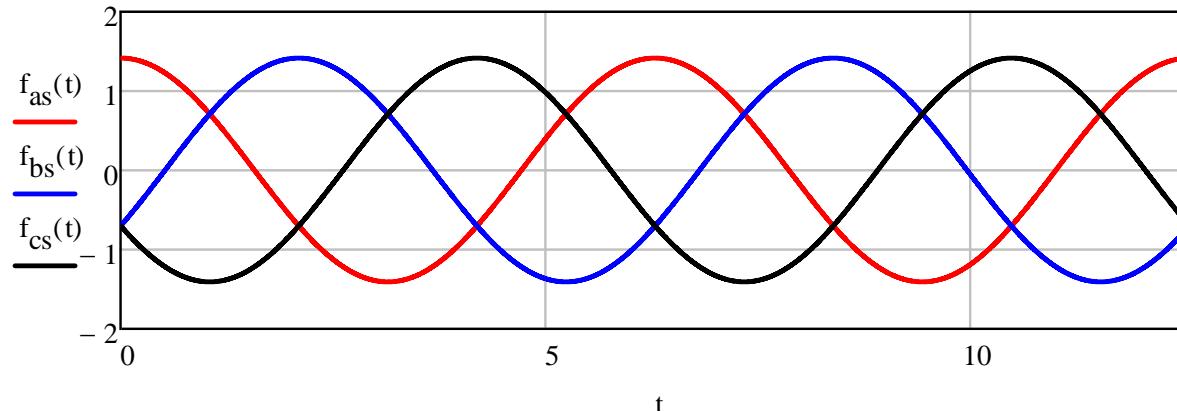
$$f_{0s} = 0 = \text{const.}$$

$$f_{\max s} = \sqrt{f_{qs}^2 + f_{ds}^2}$$

Umesto trofaznog naizmeničnog sistema dobijamo dvofazni sistem.  
Transformisane veličine se ne menjaju u vremenu.

# Statorske veličine $\omega_{rs} = \omega_s$

Na graficima  
 $\omega_s=1$



# Šta se postiže ovom transformacijom?

## Rotorske veličine

Kada je  $\omega_{rs} = \omega_s = \text{const}$ ,  $\theta_s(0) = 0$  i  $\theta_{rsr} = \theta_r = \theta_s - \theta$ ,  
za simetričan rotorski sistem:

$$f'_{ar} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0)]$$

$$f'_{br} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0) - \alpha]$$

$$f'_{cr} = f'_{\max r} \cdot \cos[(\omega_s - \omega) \cdot t + \theta_r(0) + \alpha]$$

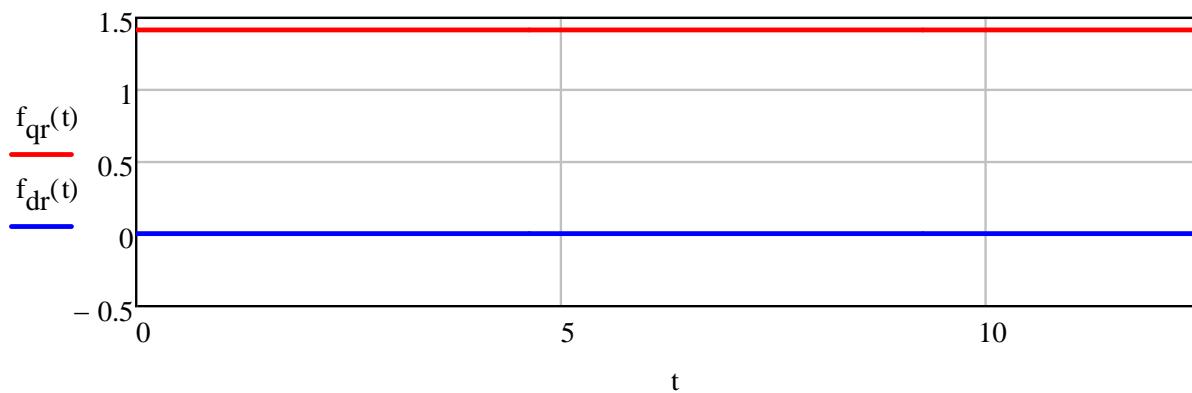
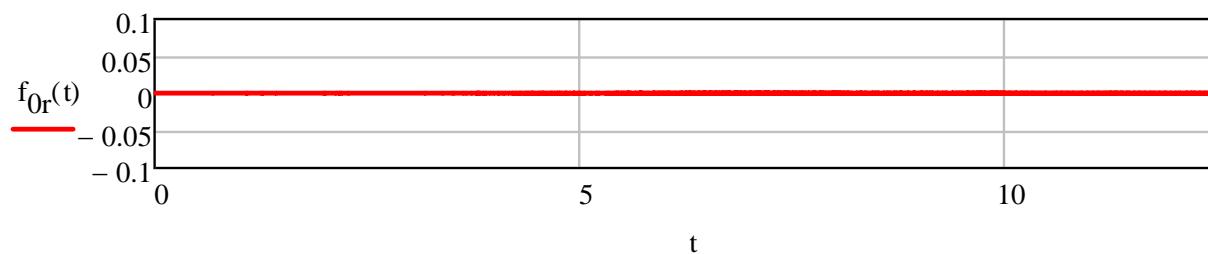
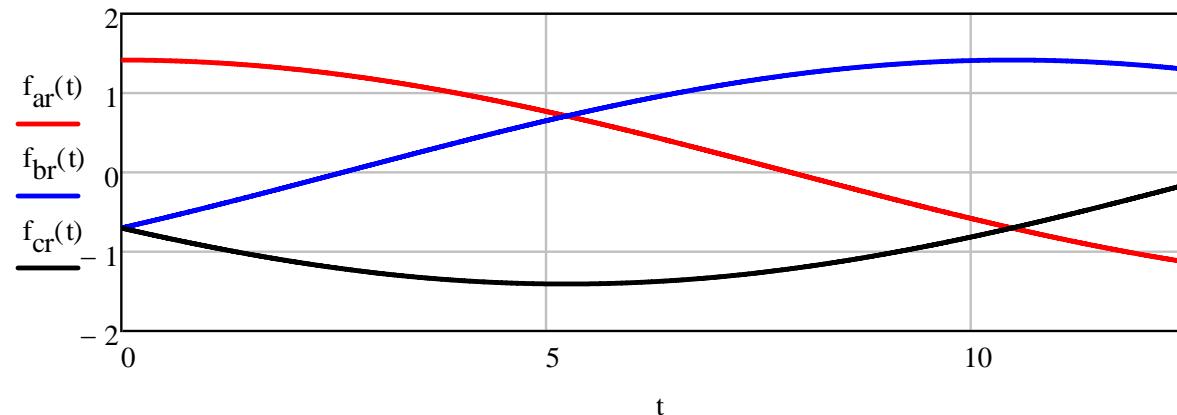
posle transformacije dobija se:

$$f'_{qr} = f'_{\max r} \cdot \cos \theta_r(0)$$

$$f'_{dr} = -f'_{\max r} \cdot \sin \theta_r(0)$$

$$f'_{0r} = 0$$

# Rotorske veličine $\omega_{rs} = \omega_s$



# TRANSFORMACIJE NAPONSKIH JEDNAČINA ASINHRONOG MOTORA

*Prvi karakterističan slučaj:*

$$\vec{u}_{abc} = \mathbf{R} \cdot \vec{i}_{abc}$$

Množeći ovu jednačinu sa desne strane sa  $\mathbf{K}$  dobija se:

$$\vec{u}_{qd0} = \mathbf{K} \cdot \vec{u}_{abc} = \mathbf{K} \cdot \mathbf{R} \cdot \vec{i}_{abc} = \mathbf{K} \cdot \mathbf{R} \cdot (\mathbf{K})^{-1} \cdot \vec{i}_{qd0}$$

Kod simetričnih sistema je:

$$\mathbf{K} \cdot \mathbf{R} \cdot (\mathbf{K})^{-1} = R \cdot \mathbf{K} \cdot \mathbf{I} \cdot (\mathbf{K})^{-1} = R \cdot \mathbf{I} = \mathbf{R}$$

Prema tome dobija se:

$$\vec{u}_{qd0} = \mathbf{R} \cdot \vec{i}_{qd0}$$

*Drugi karakterističan slučaj:*       $\vec{u}_{abc} = \frac{d}{dt} \vec{\varphi}_{abc}$

Posle množenja sa  $\mathbf{K}$  dobija se:

$$\begin{aligned}\vec{u}_{qd0} &= \mathbf{K} \cdot \vec{u}_{abc} = \mathbf{K} \cdot \frac{d}{dt} \left[ (\mathbf{K})^{-1} \cdot \vec{\varphi}_{qd0} \right] = \\ &= \mathbf{K} \cdot \frac{d}{dt} (\mathbf{K})^{-1} \cdot \vec{\varphi}_{qd0} + \mathbf{K} \cdot (\mathbf{K})^{-1} \cdot \frac{d}{dt} \vec{\varphi}_{qd0}\end{aligned}$$

ako je  $\theta_{rs} = \omega_{rs} \cdot t$ , sledi:

$$\frac{d}{dt} \left[ (\mathbf{K})^{-1} \right] = \omega_{rs} \cdot \begin{bmatrix} -\sin \theta_{rs} & \cos \theta_{rs} & 0 \\ -\sin(\theta_{rs} - \alpha) & \cos(\theta_{rs} - \alpha) & 0 \\ -\sin(\theta_{rs} + \alpha) & \cos(\theta_{rs} + \alpha) & 0 \end{bmatrix}$$

$$\mathbf{K} \cdot \frac{d}{dt} \left[ (\mathbf{K})^{-1} \right] = \omega_{rs} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \omega_{rs} \cdot \mathbf{W}$$

Konačno je:

$$\vec{u}_{qd0} = \omega_{rs} \cdot \begin{bmatrix} \varphi_d \\ -\varphi_q \\ 0 \end{bmatrix} + \frac{d}{dt} \vec{\varphi}_{qd0}$$

Da bi bilo jasnije, prethodna jednačina se može razbiti na:

$$u_q = \omega_{rs} \cdot \varphi_d + \frac{d}{dt} \varphi_q$$

$$u_d = -\omega_{rs} \cdot \varphi_q + \frac{d}{dt} \varphi_d$$

$$u_0 = \frac{d}{dt} \varphi_0$$

# Izvedene relacije primenjene na naponske jednačine asinhronog motora:

$$\begin{bmatrix} \vec{u}_{qd0s} \\ \vec{u}'_{qd0r} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{R}'_r \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_{qd0s} \\ \vec{i}'_{qd0r} \end{bmatrix} + \\ + \begin{bmatrix} \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \mathbf{W} \end{bmatrix} \begin{bmatrix} \omega_{rs} \cdot \vec{\varphi}_{qd0s} \\ (\omega_{rs} - \omega) \cdot \vec{\varphi}'_{qd0r} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \vec{\varphi}_{qd0s} \\ \vec{\varphi}'_{qd0r} \end{bmatrix}$$

$\mathbf{0}$  - kvadratna ( $3 \times 3$ ) nula matrica.

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# TRANSFORMACIJE JEDNAČINA FLUKSA ASINHRONOG MOTORA

$$\begin{bmatrix} \vec{\phi}_{qd0s} \\ \vec{\phi}'_{qd0r} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_s \cdot \mathbf{L}_s \cdot (\mathbf{K}_s)^{-1} & \mathbf{K}_s \cdot \mathbf{L}'_{sr} \cdot (\mathbf{K}_r)^{-1} \\ \mathbf{K}_r \cdot (\mathbf{L}'_{sr}) \cdot (\mathbf{K}_s)^{-1} & \mathbf{K}_r \cdot \mathbf{L}'_r \cdot (\mathbf{K}_r)^{-1} \end{bmatrix} \cdot \begin{bmatrix} \vec{i}_{qd0s} \\ \vec{i}'_{qd0r} \end{bmatrix}$$

$$\mathbf{K}_s \cdot \mathbf{L}_s \cdot (\mathbf{K}_s)^{-1} = \begin{bmatrix} \lambda_s + M & 0 & 0 \\ 0 & \lambda_s + M & 0 \\ 0 & 0 & \lambda_s + M \end{bmatrix}$$

$$M = \frac{3}{2} \cdot M_s$$

$$\mathbf{K}_r \cdot \mathbf{L}'_r \cdot (\mathbf{K}_r)^{-1} = \begin{bmatrix} \lambda'_r + M & 0 & 0 \\ 0 & \lambda'_r + M & 0 \\ 0 & 0 & \lambda'_r + M \end{bmatrix}$$

$$\mathbf{K}_s \cdot \mathbf{L}'_{sr} \cdot (\mathbf{K}_r)^{-1} = \mathbf{K}_r \cdot \mathbf{L}'_{sr} \cdot (\mathbf{K}_s)^{-1} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

U slučaju simetričnog sistema, nulta komponenta je nula u svim referentnim sistemima.

U tom slučaju naponska jednačina asinhronog motora je:

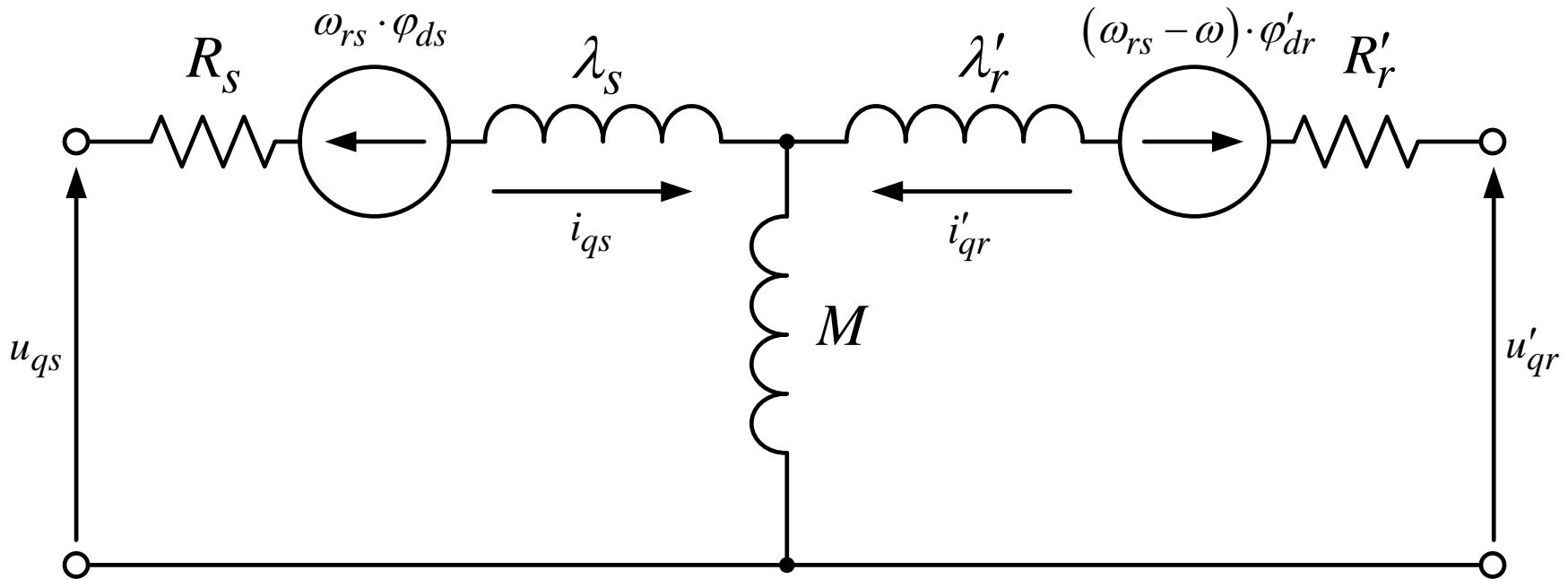
$$\begin{bmatrix} u_{qs} \\ u_{ds} \\ u'_{qr} \\ u'_{dr} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R'_r & 0 \\ 0 & 0 & 0 & R'_r \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix} +$$

$$p = \frac{d}{dt} + \begin{bmatrix} p & \omega_{rs} & 0 & 0 \\ -\omega_{rs} & p & 0 & 0 \\ 0 & 0 & p & (\omega_{rs} - \omega) \\ 0 & 0 & -(\omega_{rs} - \omega) & p \end{bmatrix} \cdot \begin{bmatrix} \varphi_{qs} \\ \varphi_{ds} \\ \varphi'_{qr} \\ \varphi'_{dr} \end{bmatrix}$$

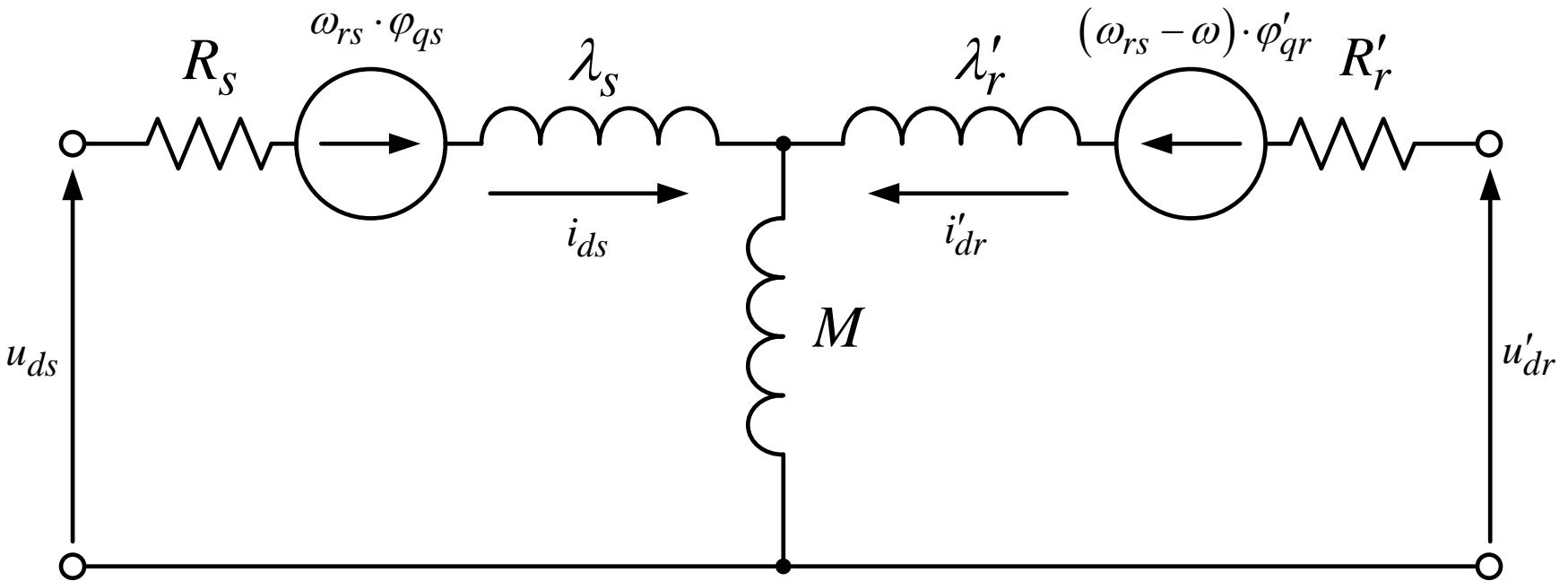
Veza između flukseva i struja je:

$$\begin{bmatrix} \varphi_{qs} \\ \varphi_{ds} \\ \varphi'_{qr} \\ \varphi'_{dr} \end{bmatrix} = \begin{bmatrix} \lambda_s + M & 0 & M & 0 \\ 0 & \lambda_s + M & 0 & M \\ M & 0 & \lambda'_r + M & 0 \\ 0 & M & 0 & \lambda'_r + M \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix}$$

# Ekvivalentna šema asinhronog motora po $q$ osi



# Ekvivalentna šema asinhronog motora po $d$ osi



Obratiti pažnju na smerove u  
generatorima "elektromotorne sile".

# JEDNAČINE MOMENTA

Ako se pođe od jednačine za moment (strana 6):

$$m_e = P \cdot \left[ (\mathbf{K}_s)^{-1} \cdot \vec{i}_{qd0s} \right]^T \cdot \frac{\partial}{\partial \theta} [\mathbf{L}'_{sr}] \cdot (\mathbf{K}_r)^{-1} \cdot \vec{i}'_{qd0r}$$

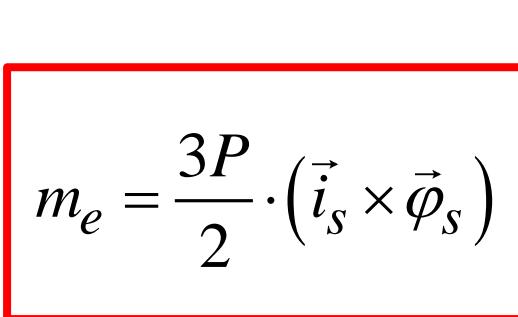
mogu se dobiti sledeći izrazi:

$$m_e = \frac{3P}{2} \cdot M \cdot (i_{qs} \cdot i'_{dr} - i_{ds} \cdot i'_{qr})$$

$$m_e = \frac{3P}{2} \cdot (\varphi'_{qr} \cdot i'_{dr} - \varphi'_{dr} \cdot i'_{qr})$$

$$m_e = \frac{3P}{2} \cdot (i_{qs} \cdot \varphi_{ds} - i_{ds} \cdot \varphi_{qs})$$

$$m_e = \frac{3P}{2} \cdot (\vec{i}_s \times \vec{\varphi}_s)$$


$$m_e = \frac{3P}{2} \cdot \frac{M}{L_r} (i_{qs} \cdot \varphi'_{dr} - i_{ds} \cdot \varphi'_{qr})$$

$$m_e = \frac{3P}{2} \cdot \frac{1}{\omega_b} (\psi'_{qr} \cdot i'_{dr} - \psi'_{dr} \cdot i'_{qr}) \quad \text{itd.}$$

# Dinamički model kavezognog asinhronog motora

Sinhrono rotirajući referentni sistem

$$\omega_{rs} = \omega_s \quad p = \frac{d}{dt}$$

---

$$u_{qs} = R_s \cdot i_{qs} + p\varphi_{qs} + \omega_{rs} \cdot \varphi_{ds} \quad (1)$$

$$u_{ds} = R_s \cdot i_{ds} + p\varphi_{ds} - \omega_{rs} \cdot \varphi_{qs} \quad (2)$$

$$0 = R'_r \cdot i'_{qr} + p\varphi'_{qr} + (\omega_{rs} - \omega) \cdot \varphi'_{dr} \quad (3)$$

$$0 = R'_r \cdot i'_{dr} + p\varphi'_{dr} - (\omega_{rs} - \omega) \cdot \varphi'_{qr} \quad (4)$$

---

$$\varphi_{qs} = L_s \cdot i_{qs} + M \cdot i'_{qr} \quad (5) \qquad \Rightarrow L_s = M + \lambda_s$$

$$\varphi_{ds} = L_s \cdot i_{ds} + M \cdot i'_{dr} \quad (6)$$

$$\varphi'_{qr} = L'_r \cdot i'_{qr} + M \cdot i_{qs} \quad (7) \qquad \Rightarrow L'_r = M + \lambda'_r$$

$$\varphi'_{dr} = L'_r \cdot i'_{dr} + M \cdot i_{ds} \quad (8)$$

---

$$m_e = \frac{3}{2} \cdot P \cdot \frac{M}{L'_r} \cdot (i_{qs} \cdot \varphi'_{dr} - i_{ds} \cdot \varphi'_{qr}) \quad (9)$$

# Dinamički model kavezognog asinhronog motora

Stacionarni referentni sistem

$$\omega_{rs} = 0 \quad p = \frac{d}{dt}$$

---

$$u_{qs} = R_s \cdot i_{qs} + p\varphi_{qs} + 0 \cdot \varphi_{ds} \quad (1)$$

$$u_{ds} = R_s \cdot i_{ds} + p\varphi_{ds} - 0 \cdot \varphi_{qs} \quad (2)$$

$$0 = R'_r \cdot i'_{qr} + p\varphi'_{qr} + (0 - \omega) \cdot \varphi'_{dr} \quad (3)$$

$$0 = R'_r \cdot i'_{dr} + p\varphi'_{dr} - (0 - \omega) \cdot \varphi'_{qr} \quad (4)$$

---

$$\varphi_{qs} = L_s \cdot i_{qs} + M \cdot i'_{qr} \quad (5) \qquad \Rightarrow L_s = M + \lambda_s$$

$$\varphi_{ds} = L_s \cdot i_{ds} + M \cdot i'_{dr} \quad (6)$$

$$\varphi'_{qr} = L'_r \cdot i'_{qr} + M \cdot i_{qs} \quad (7) \qquad \Rightarrow L'_r = M + \lambda'_r$$

$$\varphi'_{dr} = L'_r \cdot i'_{dr} + M \cdot i_{ds} \quad (8)$$

---

$$m_e = \frac{3}{2} \cdot P \cdot \frac{M}{L'_r} \cdot (i_{qs} \cdot \varphi'_{dr} - i_{ds} \cdot \varphi'_{qr}) \quad (9)$$

# Ne smemo zaboraviti Njutnovu jednačinu

$$J \frac{d\omega_m}{dt} = m_e - m_m$$

$$\omega_m = \frac{1}{P} \cdot \omega$$

Njutnova jednačina je ista u bilo kom referentnom sistemu.

# NORMALIZACIJA

Potrebno je na već poznate bazne vrednosti dodati:

$$U_{qdb} = U_{s \max fazno} = \sqrt{2} \cdot U_b$$

$$I_{qdb} = I_{s \max fazno} = \sqrt{2} \cdot I_b$$

$$P_b = (3/2) \cdot U_{qdb} \cdot I_{qdb}$$

$$\varphi_b = \frac{U_{qdb}}{\omega_b}$$

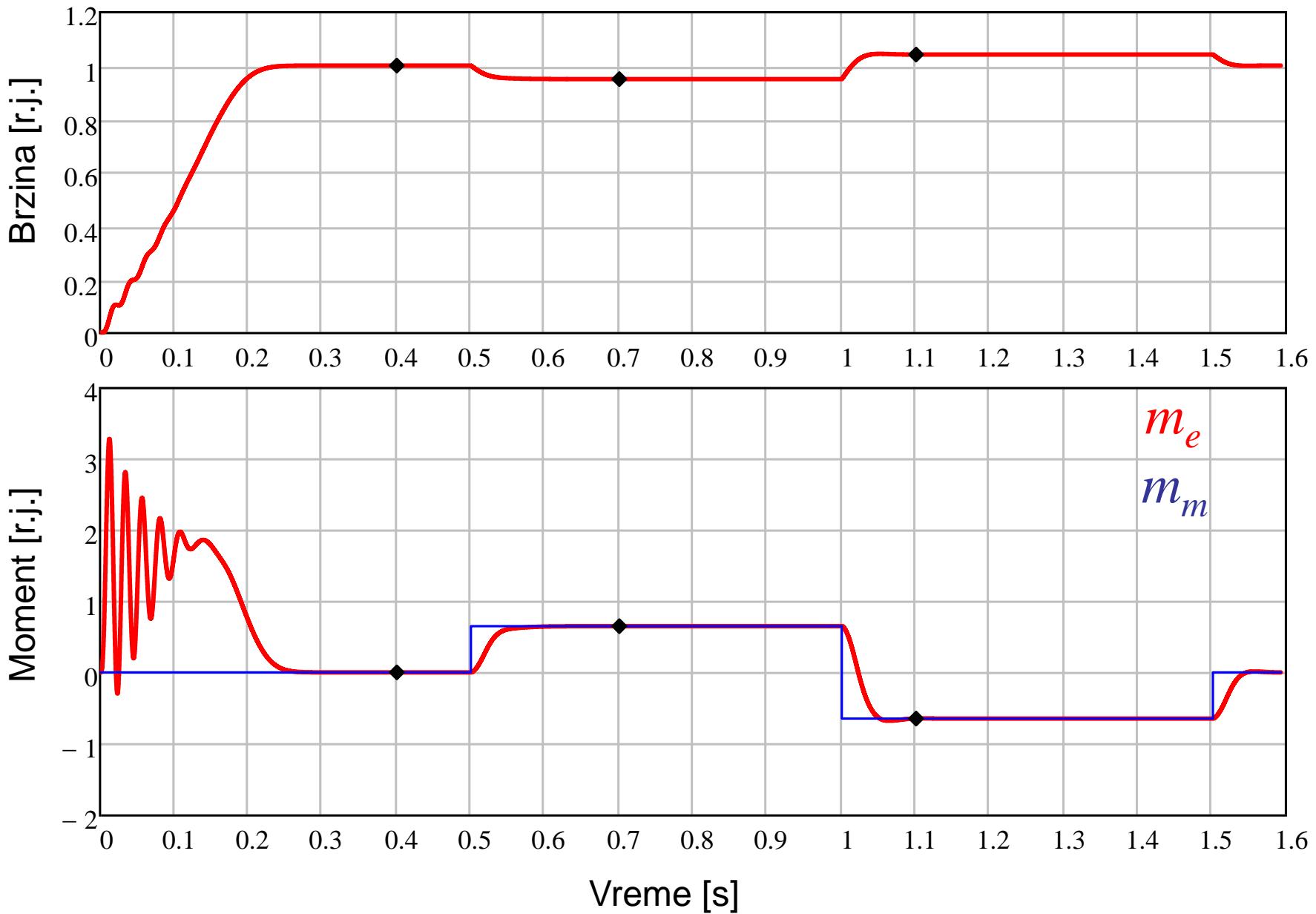
Sve ostale jednačine se normalizuju na uobičajeni način.

# Prelazni procesi

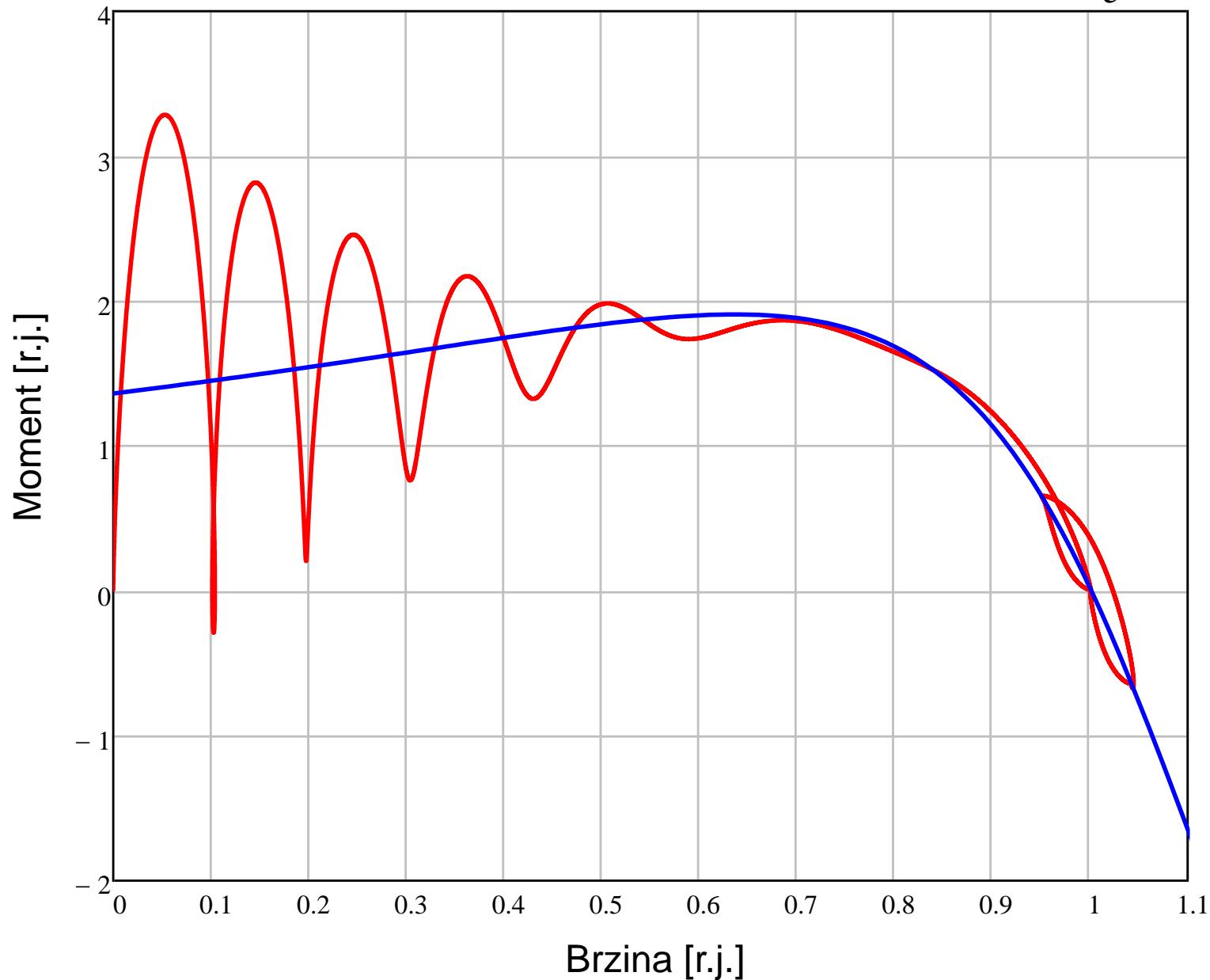
Start motora u praznom hodu, promene opterećenja

- Vremenski dijagrami momenta i brzine
- Vremenski dijagrami promene faznih struja statora i rotora
- Mehanička karakteristika ( $m_e(\omega)$ )
- Vremenski dijagram promene  $qd$ -komponenti statorskih i rotorskih struja i flukseva
- Dijagrami prostornih vektora statorske i rotorske struje, statorskog i rotorskog fluksa

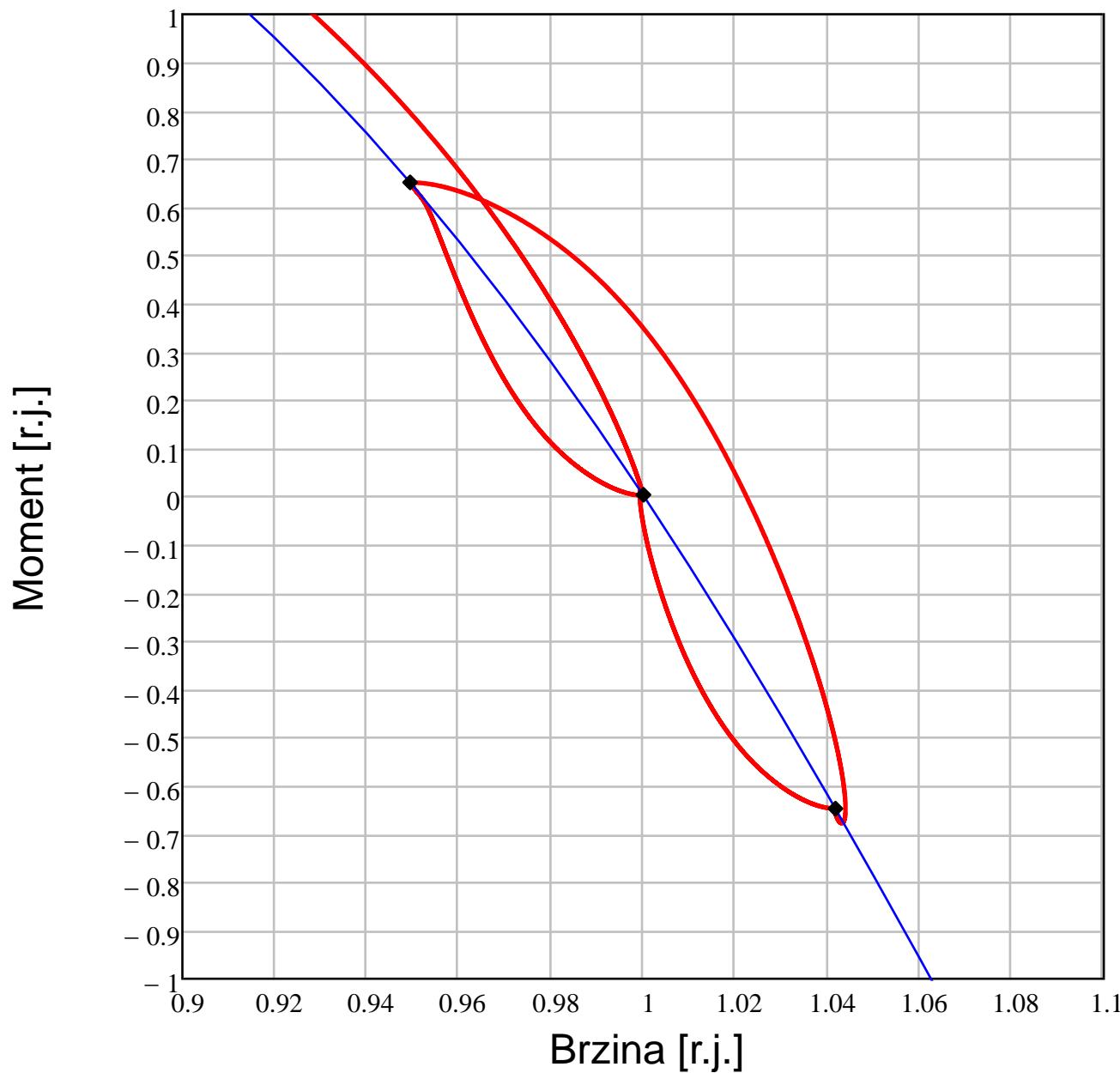
# Vremenski dijagram brzine i momenta



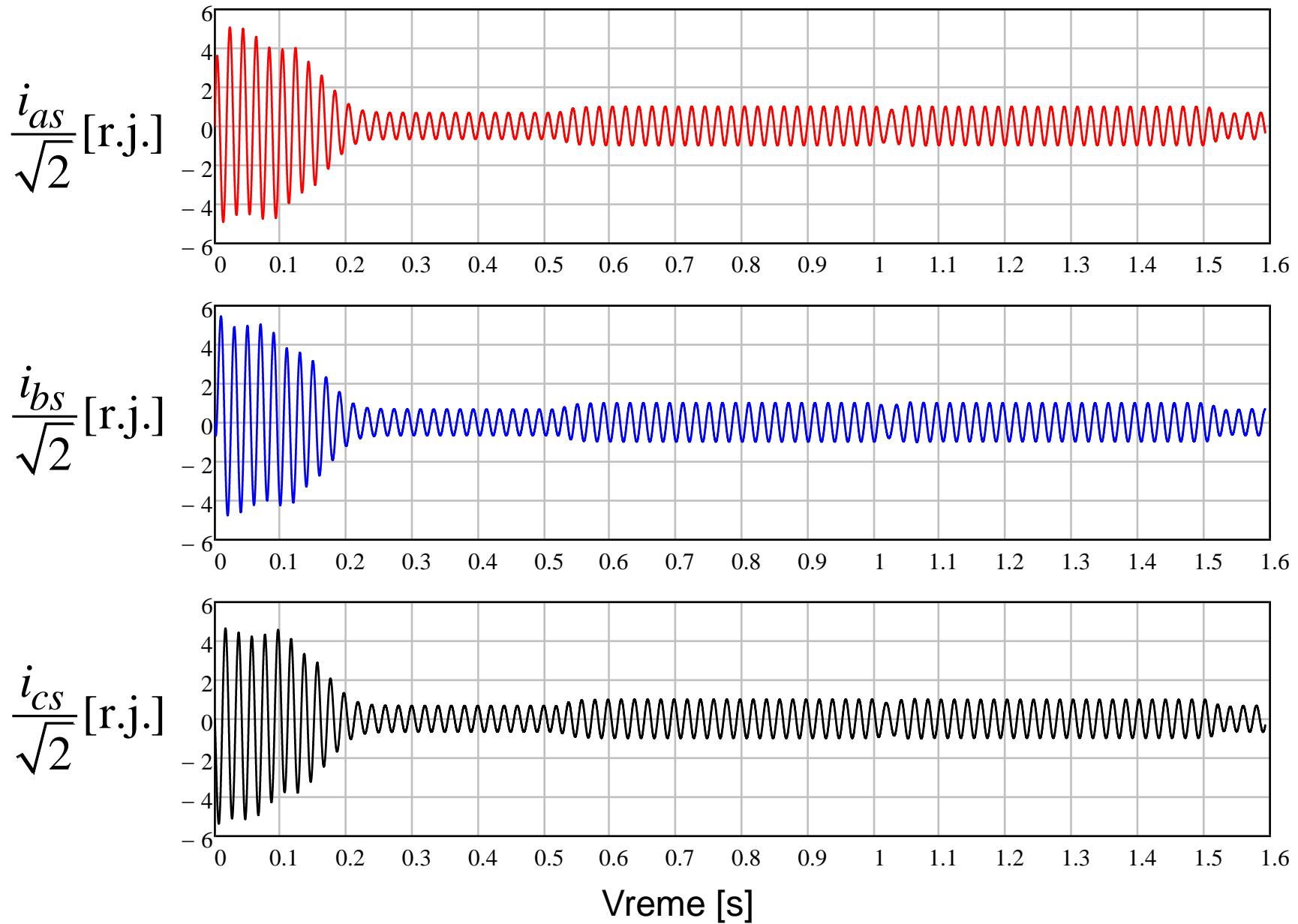
# Statička karakteristika i dijagram $m_e(\omega)$



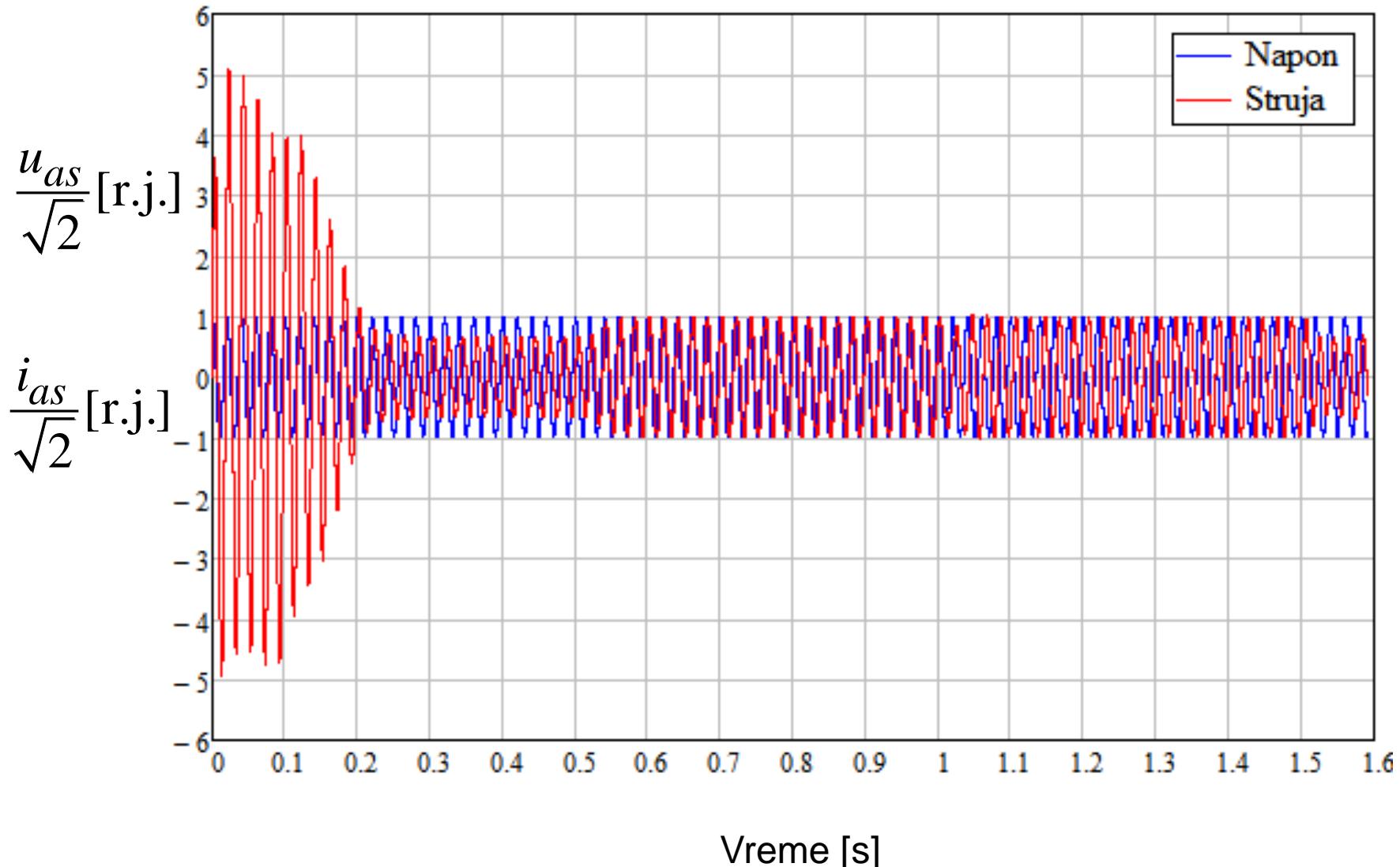
# Statička karakteristika i dijagram $m_e(\omega)$



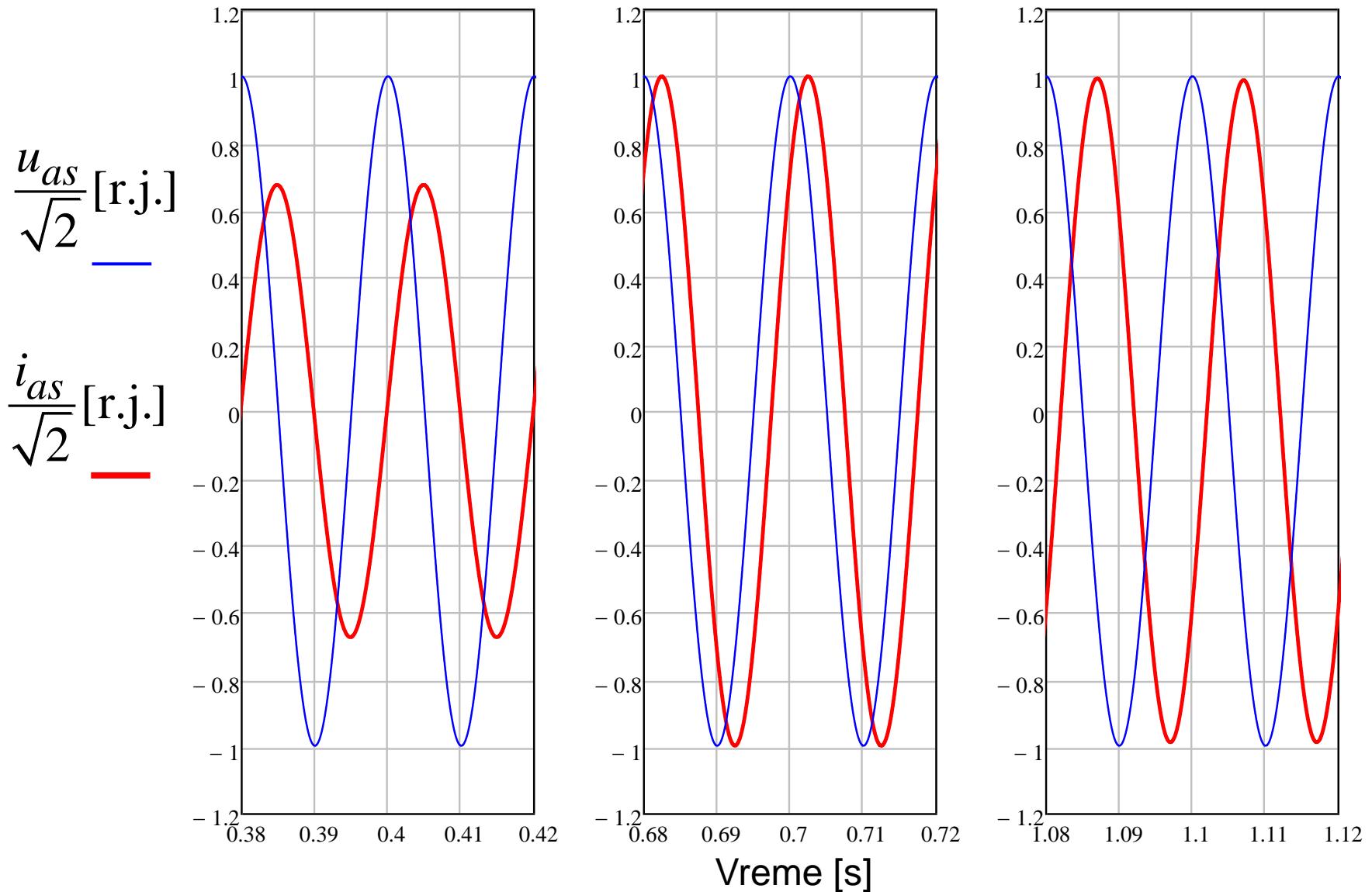
# Vremenski dijagrami statorskih struja



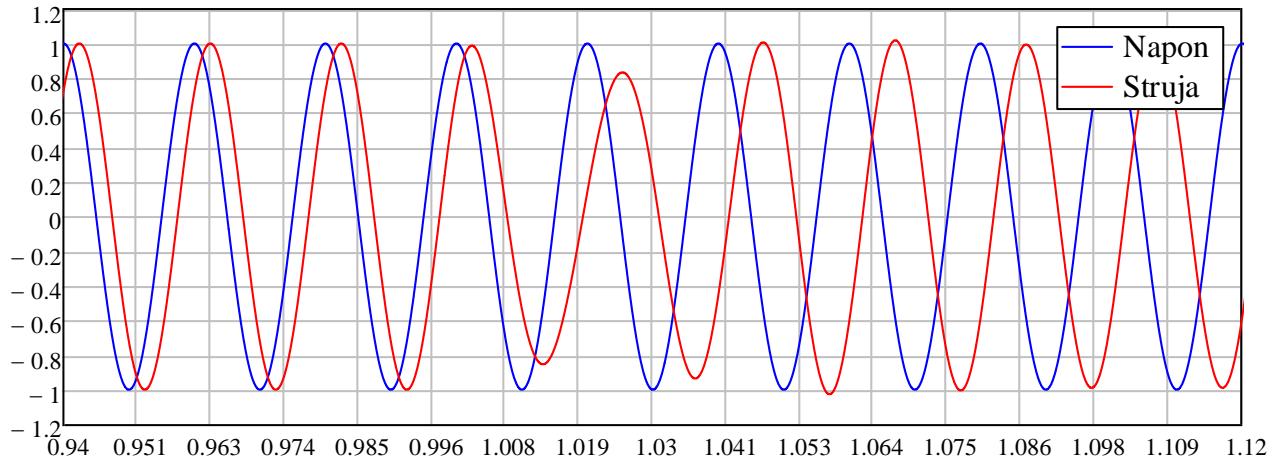
# Vremenski dijagrami statorskog faznog napona i struje



# Vremenski dijagrami statorskog faznog napona i struje

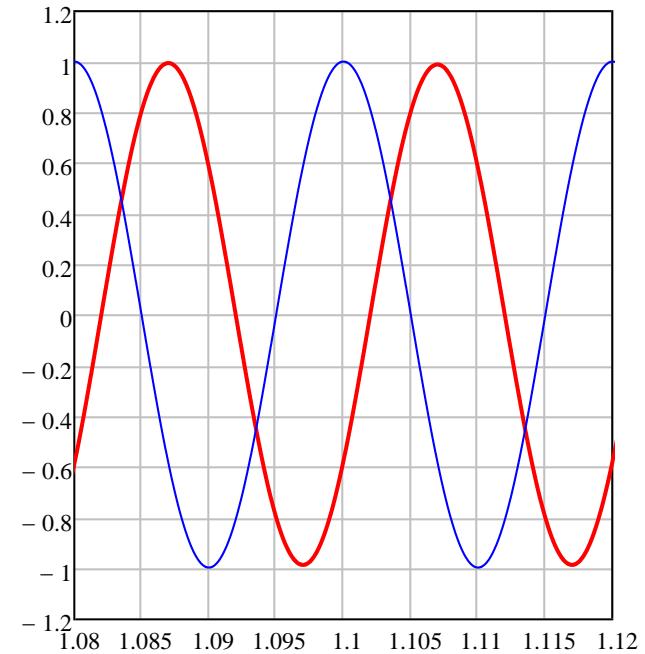
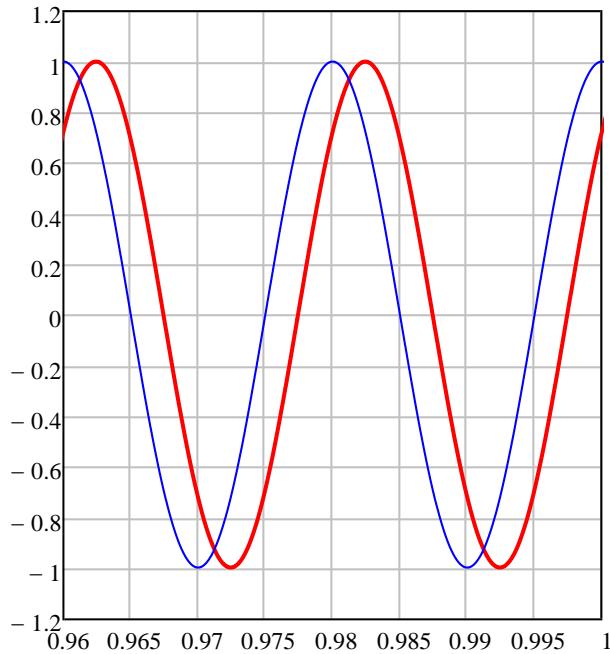


# Vremenski dijagrami statorskog faznog napona i struje



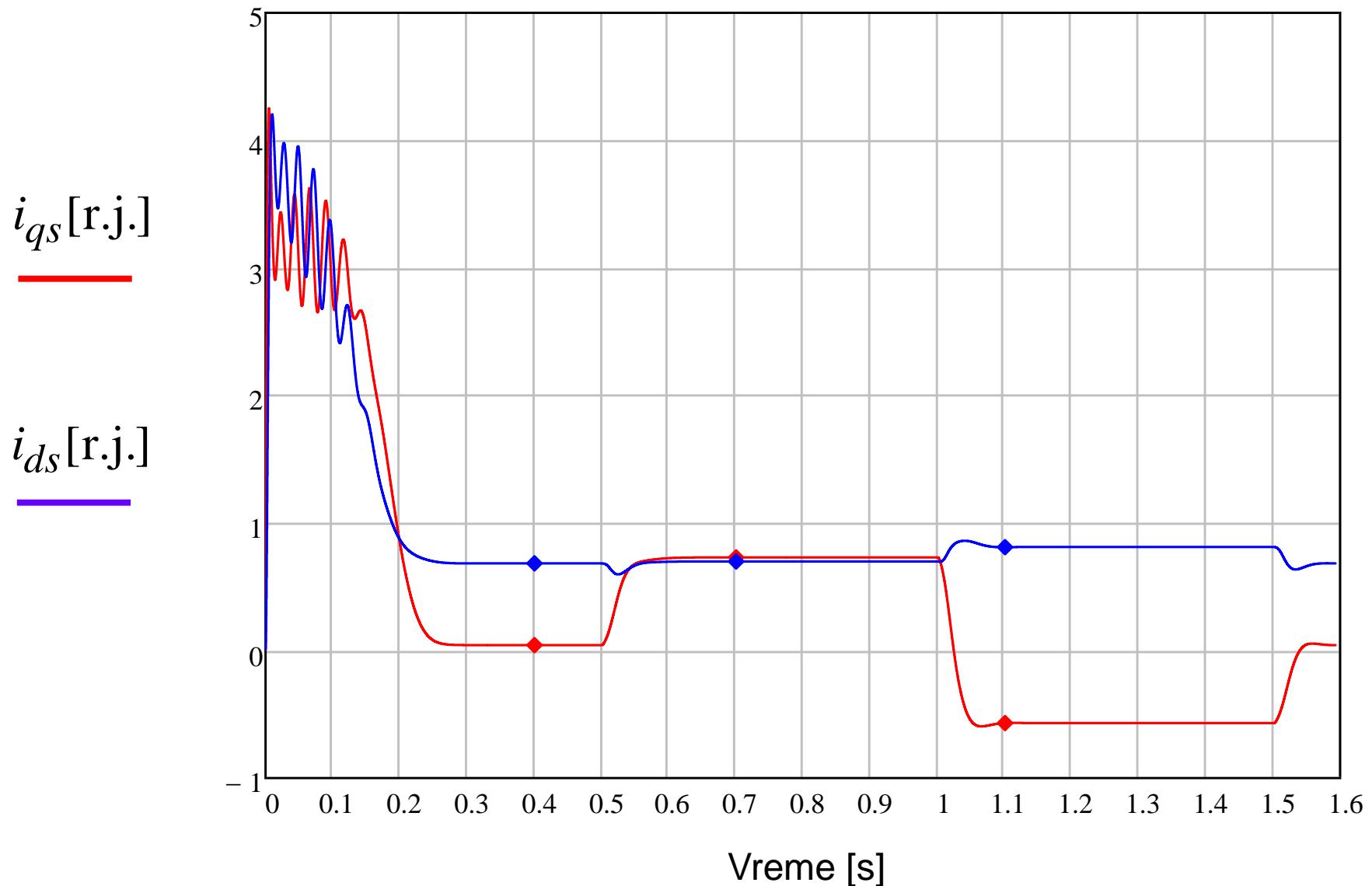
$$\frac{u_{as}}{\sqrt{2}} \text{ [r.j.]}$$

$$\frac{i_{as}}{\sqrt{2}} \text{ [r.j.]}$$

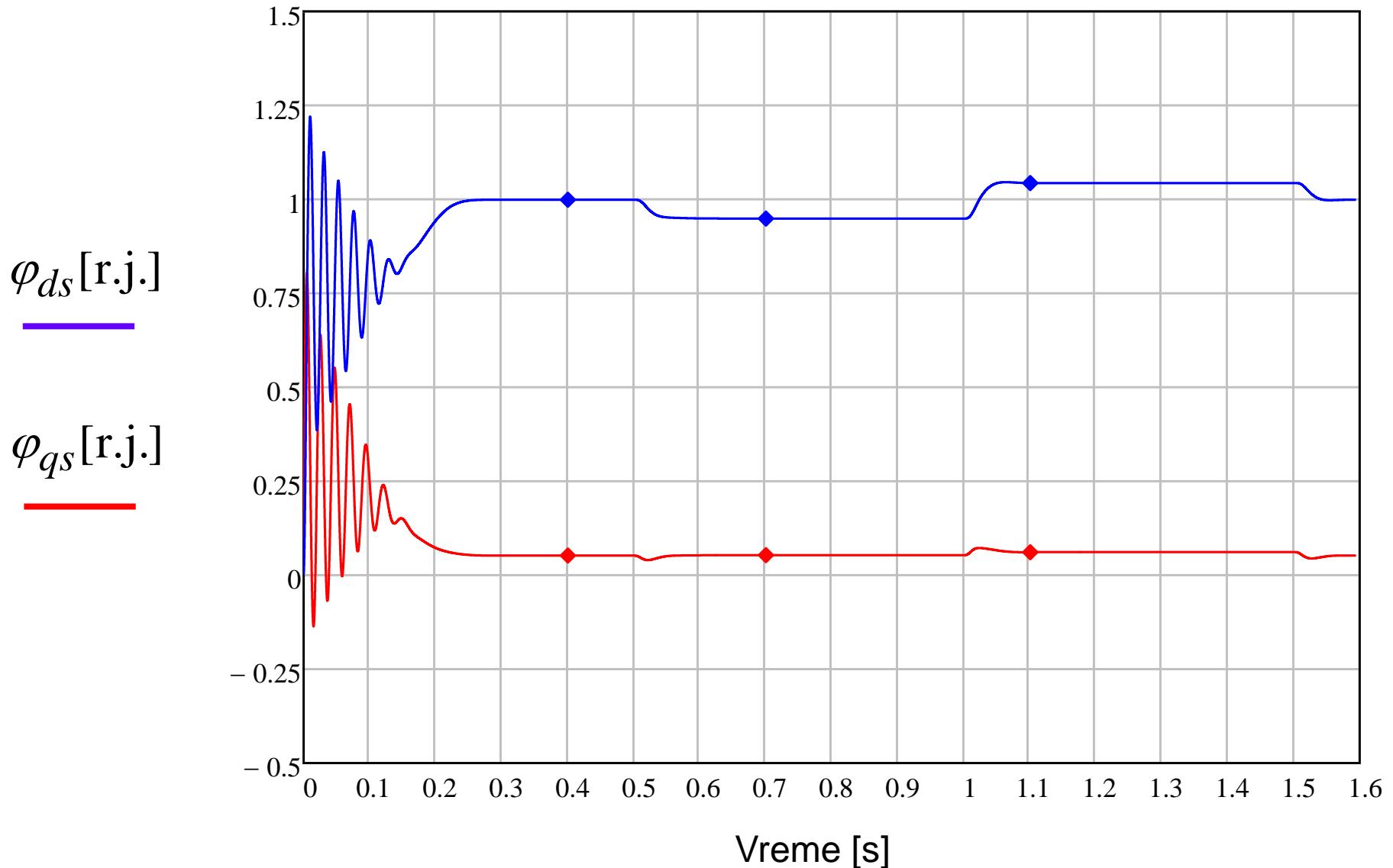


Vreme [s]

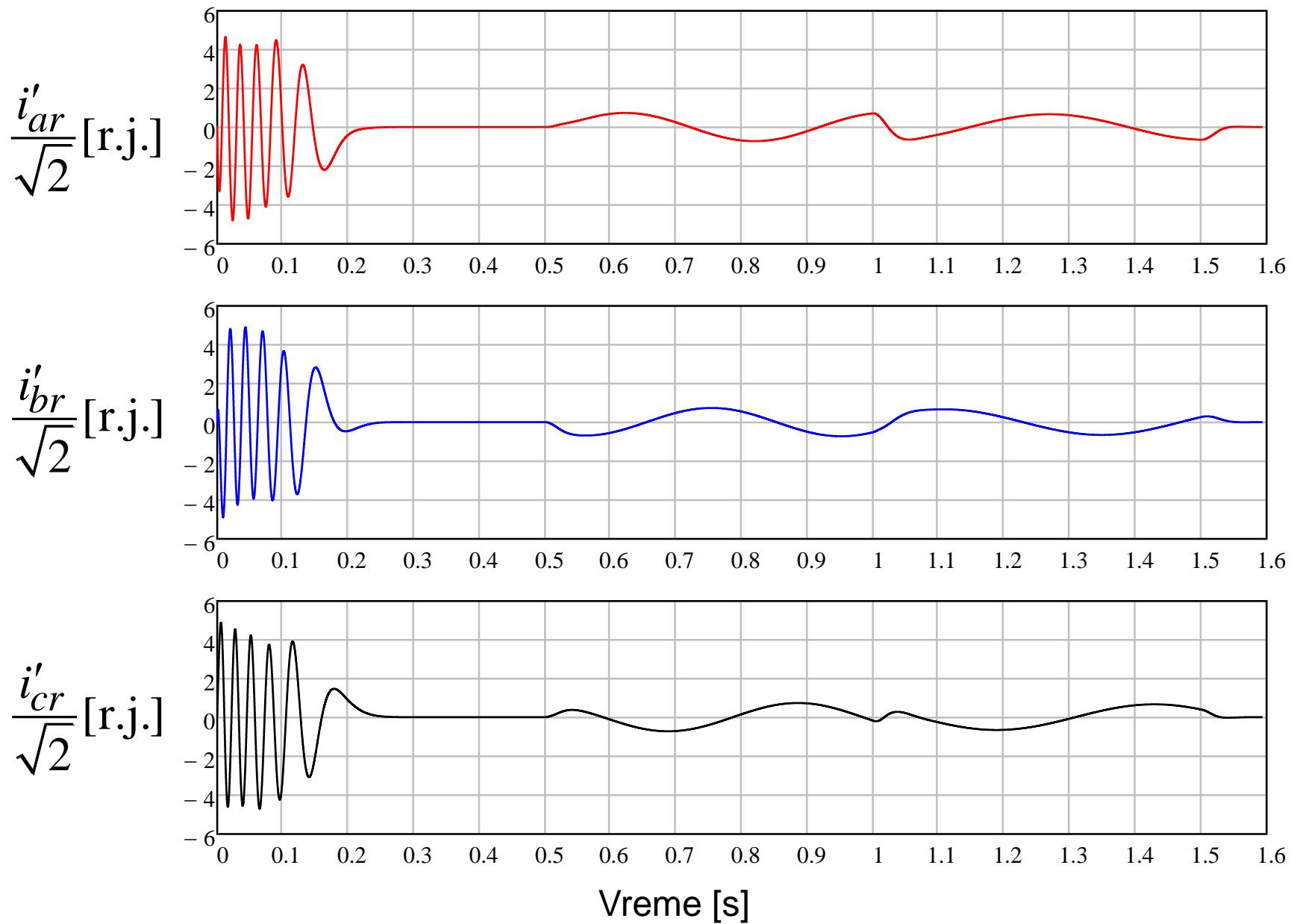
# Vremenski dijagrami q i d komponente statorske struje



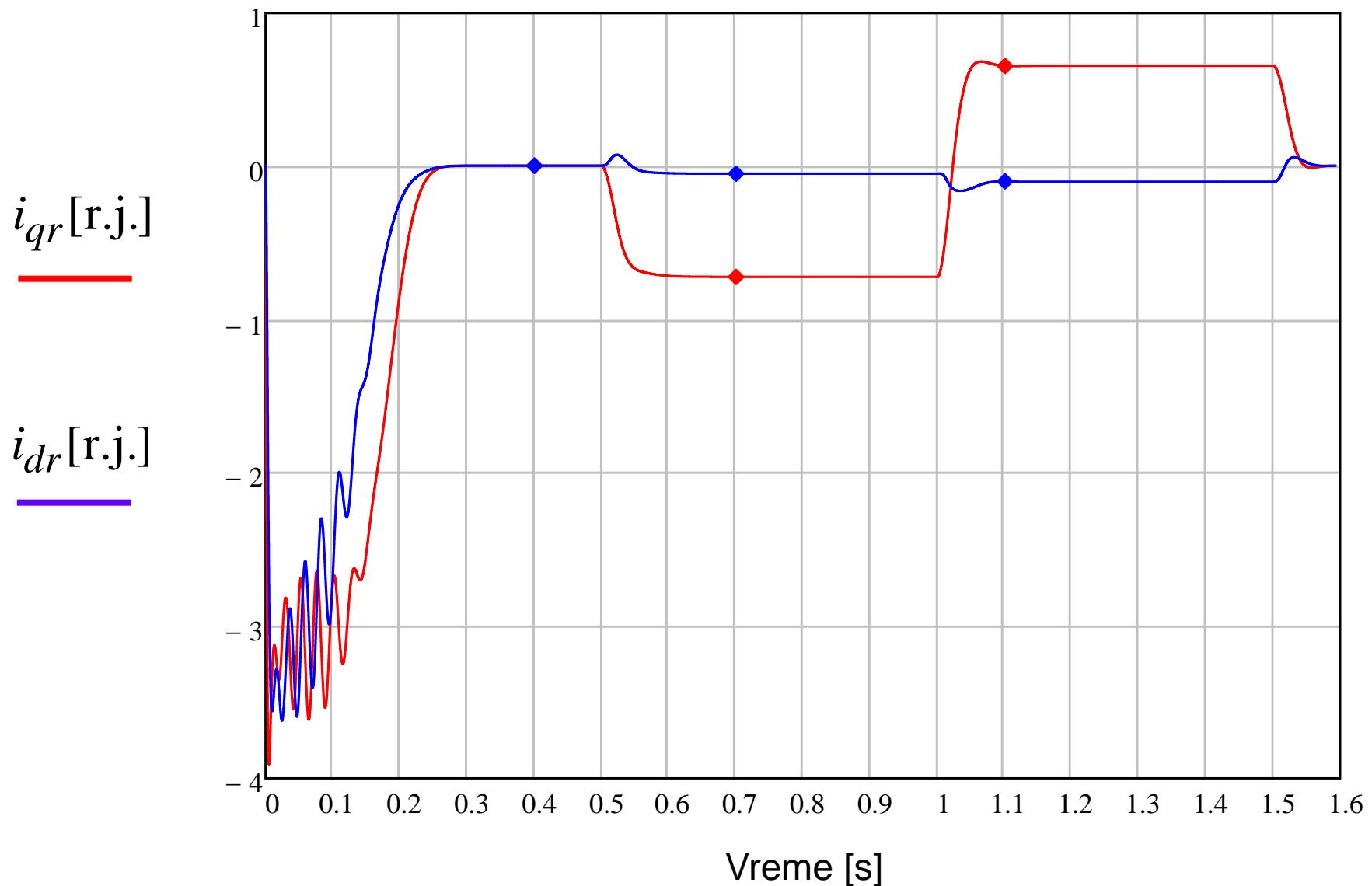
# Vremenski dijagrami q i d komponente statorskog fluksa



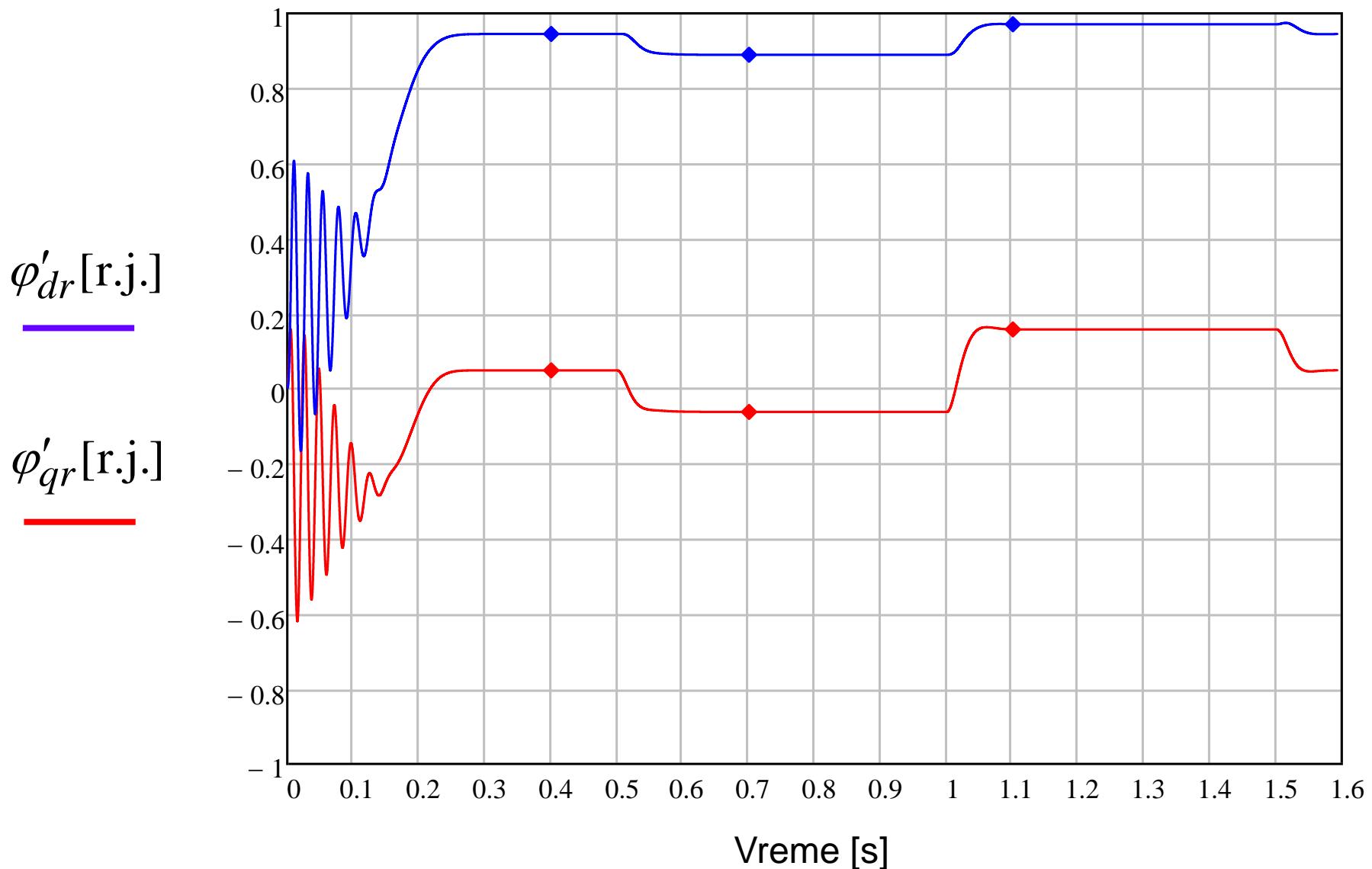
# Vremenski dijagrami rotorskih struja



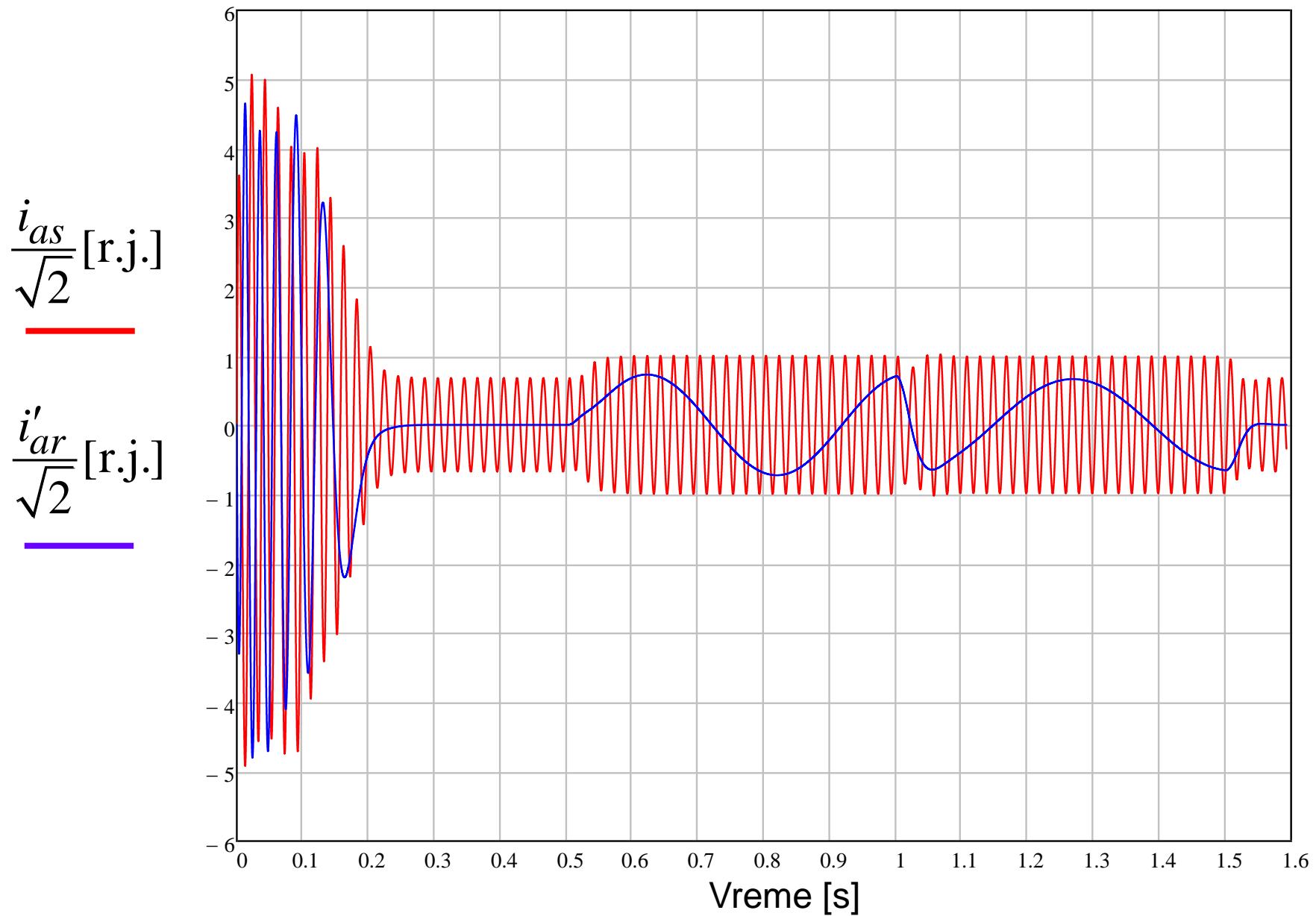
# Vremenski dijagrami q i d komponente rotorske struje



# Vremenski dijagrami q i d komponente rotorskog fluksa



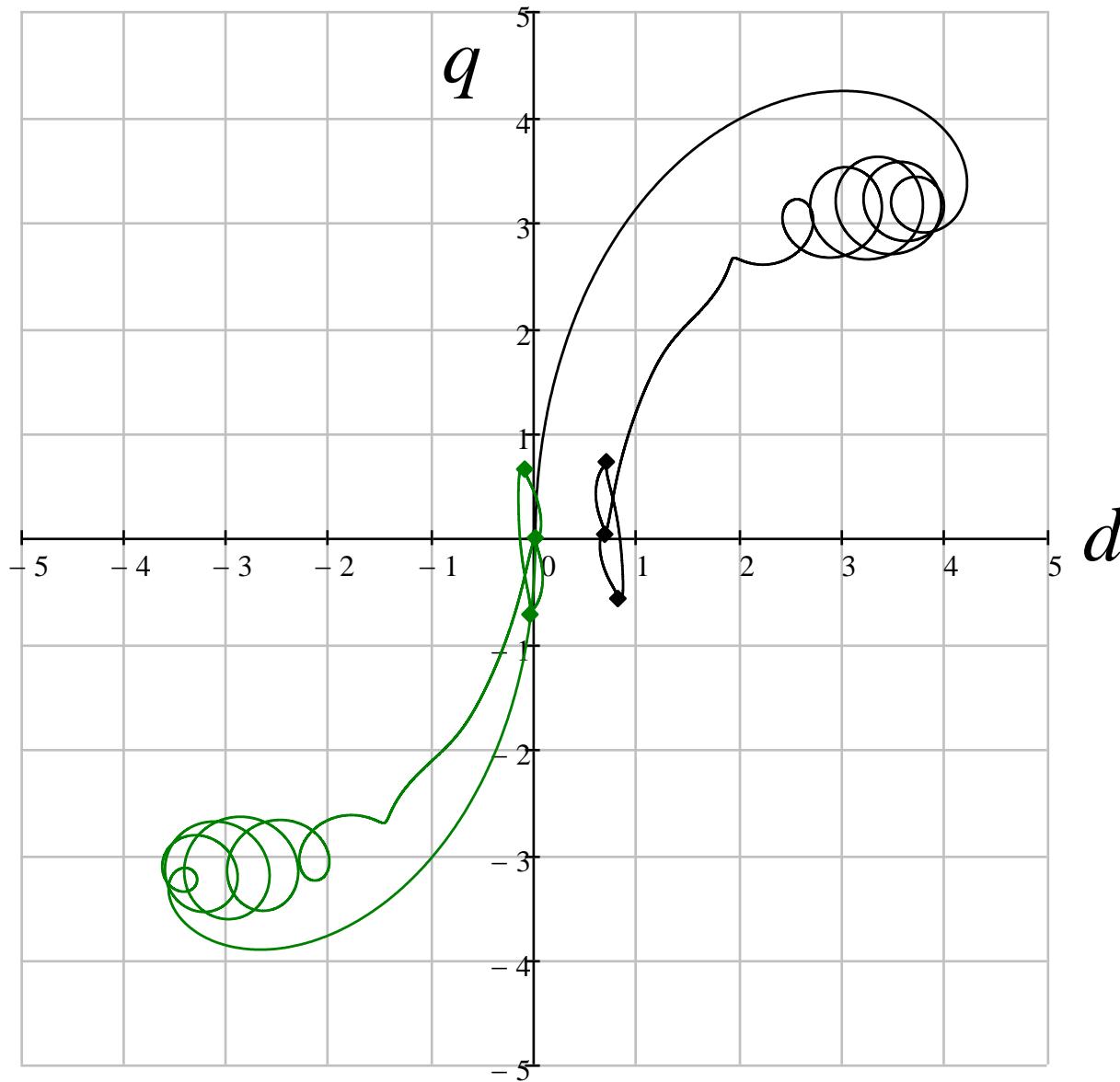
# Vremenski dijagram statorske i rotorske struje



# Dijagrami prostornih vektora statorske i rotorske struje

$\vec{i}_s$  [r.j.]

$\vec{i}'_r$  [r.j.]



# Dijagrami prostornih vektora statorskog i rotorskog fluksa

$\vec{\phi}_s$  [r.j.]

$\vec{\phi}'_r$  [r.j.]

